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Estimating the Planck´s Constant, an undergraduate experiment with STEM approach

Estimando la Constante de Planck, un experimento de pregrado con enfoque STEM

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ABSTRACT:

One of the permanent concerns in teaching at the university level is that the student is able to understand the concepts taught in the different courses and create correlations between them that allow the construction of solid and interdisciplinary knowledge. The STEM (Science, Technology, Engineering, and Mathematics) methodology allows students to connect ideas, develop experimental procedures and implement computational simulations to solve specific problems in the different fields of science and engineering. In this work, the STEM methodology is used to experimentally estimate the value of Planck's constant, based on problem-based learning that involves not only the theoretical study of semiconductor physics using the principle of operation of a light-emitting diode (LED) but also allows the development of computational and scientific programming skills by using the Arduino platform as a tool for data acquisition and analysis. In the proposal presented here, an Arduino Uno is used to control the current flowing through an LED, thus allowing to build a characteristic curve of current (I) vs. voltage (V) and thus determine the value of the bandgap of the semiconductor that makes up the P-N junction of the LED. If LEDs with emission at different wavelengths are used, it is possible to determine by means of a linear fit the value of the Planck's constant. The results obtained are promising, as long as this is understood as a low-cost system that allows having errors of around 4% with respect to the values obtained in the literature.

Keywords: STEM, Planck´s Constant, LED

RESUMEN:

Una de las preocupaciones permanentes en la enseñanza a nivel universitario, es que el estudiante logre comprender los conceptos impartidos en los distintos cursos y crear correlaciones entre ellos que permitan la construcción de un conocimiento sólido e interdisciplinar. La metodología STEM Science, Technology, Engineering and Mathematics) permite que los estudiantes logren conectar ideas, desarrollar procedimientos experimentales e implementar simulaciones computacionales que permitan resolver problemas específicos en los distintos campos de la ciencia y la ingeniería. En este trabajo se utiliza la metodología STEM para estimar con mediciones experimentales el valor de la constante de Planck, partiendo de un aprendizaje basado en problemas y que involucra no solo el estudio teórico de la física de semiconductores al usar el principio de funcionamiento de un LED sino que permite el desarrollo de habilidades computacionales y de programación científica al usar la plataforma Arduino como una herramienta para la adquisición y el análisis de datos. En la propuesta que se presenta aquí, se utiliza un Arduino Uno para controlar la corriente que circula a través de un LED, permitiendo así construir una curva característica de corriente (I) vs. Voltaje (V) y determinar así el valor de la banda prohibida del semiconductor que conforma la unión P-N del LED. Si se usan LED's con emisión en distintas longitudes de onda, es posible determinar por medio de un ajuste lineal el valor de la constante de Planck. Los resultados obtenidos son prometedores, toda vez que se entienda este como un sistema de bajo costo y que permita tener errores alrededor del 4% respecto a los valores obtenidos en la literatura.

Palabras clave: STEM, Constante de Planck, LED

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1. Introduction

Nowadays, a new focus on science and engineering education has taken importance. Particularly, the STEM (acronym for Science, Technology, Engineering, and Mathematics) method has been adopted in a lot of countries and institutions, both basic and university education. In this learning model, based on problem-solving, students can understand a real-world phenomenon from the direct interaction between theory, experimentation, and the use of computational models. [1]. An advantage to adopting those new approaches today is the availability of technological devices of easy access and programming for users, and with very low cost as the case of Arduino Cards [2]. Some interesting examples showing applications of Arduino to experimental teaching in physics can be found [3-5]. In this perspective, it is possible to propose from the STEM methodology the study of phenomena that are of utmost importance in the field of science and that allow building solid knowledge in the student who advances courses at the university level. A problem that has become important in the area of Quantum Mechanics is the experimental determination of physical constants such as Planck´s constant because it appears not only in physics models and problems but also in chemistry, material science, semiconductor science and technology, and in general, in all disciplines related to atomic and molecular fundamental and applied sciences. As a new metrological conception, the General Conference on Weights and Measures (CGPM) at its 26th meeting in November 2018 approved a the revision of the SI, in which seven fundamental constants taken as exact, including the Planck´s constant, would be the base of the system. The starting date to implement these changes was on 20 May 2019 [6]. Normally, to measure a fundamental constant requires having expensive and relatively complex instrumentations. In the case of introductory university courses for science and engineering, one of the most known methods consists in using the extrinsic photoelectric effect, in which a Hg lamp, different interference filters, a photocell, a nano ammeter or nano voltmeter, as well as lenses, diaphragms and an optical bench, are required. The cost of this instrumentation is significant. A relatively easy to do method for Planck´s constant estimation using commercial Light Emitting Diodes (LEDs), a very simple circuit, two multimeters, and a laboratory spectrometer, was reported a time ago [7]. Since that report, its procedure has been reproduced only including some instrument improvements [8-10]. However, the data analysis method used in those reports has received serious criticism because a set of semiconductor properties as diffusion length of minority carriers, the diffusion constant, and the density of the minority carriers are not considered in the model used [11, 12].

In this work, Planck´s constant estimation in the context of a STEM method approach is reported. An Arduino Uno card was implemented to simultaneously measure the electrical current in a LED and the voltage between its terminals. In comparison to the revised literature, this work has some educative novelty features: 1. Involves semiconductor physical concepts as band gap, semiconductor junction, and band model as well as spectroscopic ones as intra-band transitions and emission spectrum of solids; 2. Uses a proper physical model for the current-voltage response of a LED as suggested by Morehouse [12]; 3. Incorporates STEM strategies, and; 4. Introduces current metrological rules and concepts to the uncertainties analysis.

2. Models, Experimental and Analysis

2.a. A brief of the physics of a LED

It is well known that the current I and the voltage V in a LED satisfies Equation (1):

$$I(V) = A\left(e^{\frac{e}{\eta kT}(V-V_g)} - 1\right) \quad (1)$$

in which e, k, and T stand for the electron charge, the Boltzmann´s constant, and the absolute temperature. The dimensionless constant η takes into account the charge diffusion across the diode junction and the electrons and holes recombination in the junction region that produces the current, and its values can be between 1 (only for diffusion) and 2 (only for recombination) [12]. V_g is the V value as multiplied with e to quantify the band gap of the semiconductor E_g. A is a constant having current unities. When the LED begins to emit, 1 is smaller than the exponential term in Equation (1) because kT/e is of the order of 26 mV at 300 K (room temperature) while V-V_g can be around 1 V, hence, kT/η $e(V$-$V_g)$ ≈ 60. In this way, Equation (1) can be approximated to Equation (2):

$$I(V) = Ae^{\frac{e}{\eta kT}(V-V_g)} \quad (2)$$

Measurements of I and V could be used to obtain the V_g value. This is possible by mean of a data transformation consisting in plotting the natural logarithm of I versus V, that is,

$$Ln(I) = \left(\frac{e}{\eta kT}\right)V - \left(\frac{eV_g}{\eta kT}\right) + Ln(A) \quad (3)$$

There is a region in which $Ln(I)$ versus V has a linear behaviour, then $e/\eta kT$ and $e\,V_g/\eta kT$ would be its slope m and intercept b, respectively, if $Ln(A)$ is neglected in Equation (3). This could be explained as following: A is a constant proportional to I_0, the saturation current. As it is well known, typically I_0 can be 1.53×10^{-21} A and 2.23×10^{-11} A for a green LED and for an infrared LED, respectively [12]. In this way, $ln(A)$ can be negligible as compared to $ln(I)$, hence, estimating m and b from the experimental data, V_g could be estimated as

$$V_g = \frac{b}{m} \quad (4)$$

On the other hand, if a proper voltage is applied to a LED, electrons in the valence band absorb energy from the external electrical field and promote it to the conduction band. Then, the electrons return to the valence band emitting photons. In general, a larger number of electrons emits photons for the transition between the bottom of the conduction band to the top of the valence band. Under this assumption, from the emission spectrum of a LED, the wavelength of the maximum can be related to the bandgap of the semiconductor by Equation (5):

$$E_g = \left(\frac{c}{\lambda}\right) h \quad (5)$$

where h is Planck´s constant, λ the wavelength in the maximum intensity emitted by the LED, and c the speed of light in vacuum.

2.b. Experimental procedure

Measurements of current and voltage across the LEDs were carried out by using an Arduino Uno card connected as shown in Fig. 1. P is a 1kΩ potentiometer, R is a 330 Ω resistance, L is a LED and the green lines represent connection cables. UC is the USB card connection to the PC. This connection works both as a power supply for the card and as the acquisition data port. Analog ports A0, A1, and A2 must be programmed as lecture ones. The power supply for the circuit is taken from the 5 V PIN and the reference is the GND PIN.

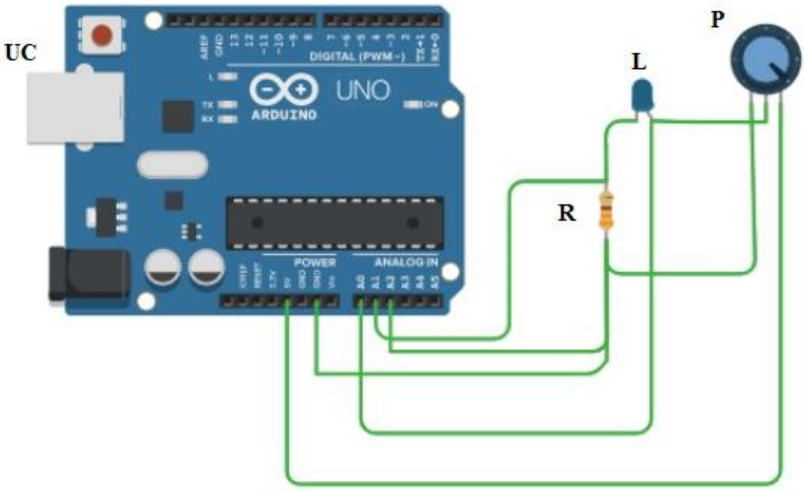

Fig.1. Arduino Uno card and the circuit used to measure the current versus voltage curve of each LED. The figure was simulated by the free application Tinkecard [13].

After a simulation using the Tinkecard platform [13], extracting the I-V data and plotting it, the physical circuit was constructed and data from five different LEDs (orange, yellow, red, green, and blue), were obtained. In both cases, a code written in Arduino language adapted from that of reference [5], was implemented. The codes allow exporting the I-V data as .txt file readable in any plotter program. In this work, Excel from Microsoft application was used.

The emission spectrum for each LED was obtained by an Ocean optics USB 4000 spectrometer, and the results are shown in Fig. 3. A smartphone as explained in *Spectral Worldbench* [14] could replace the spectrometer. It could be a better option not only because can strength the STEM strategies for this project but also because can be more proper in COVID-19 pandemic times.

2.c. Results

In Fig. 2 A., an example of the I-V curve as obtained from the Arduino Uno card with the setup illustrated in Fig.1, is shown. In the same figure, B, C, D, E, and F, show plots of Ln (I) versus V in their respective linear region for the orange, yellow, red, green, and blue LEDs, respectively. Inset can be seen in the respective linear fitting equation. From those equations, the data are shown in the second and the third columns in Table 1, were calculated. Hence, the corresponding V_g value for each LED (the last column in Table 1.), were estimated. In addition, the wavelength corresponding to the maximum emission of the different LEDs studied in this work, as well as the respective photon frequencies are presented in Table 1. The wavelength values were obtained from the emission spectra shown in Fig. 3.

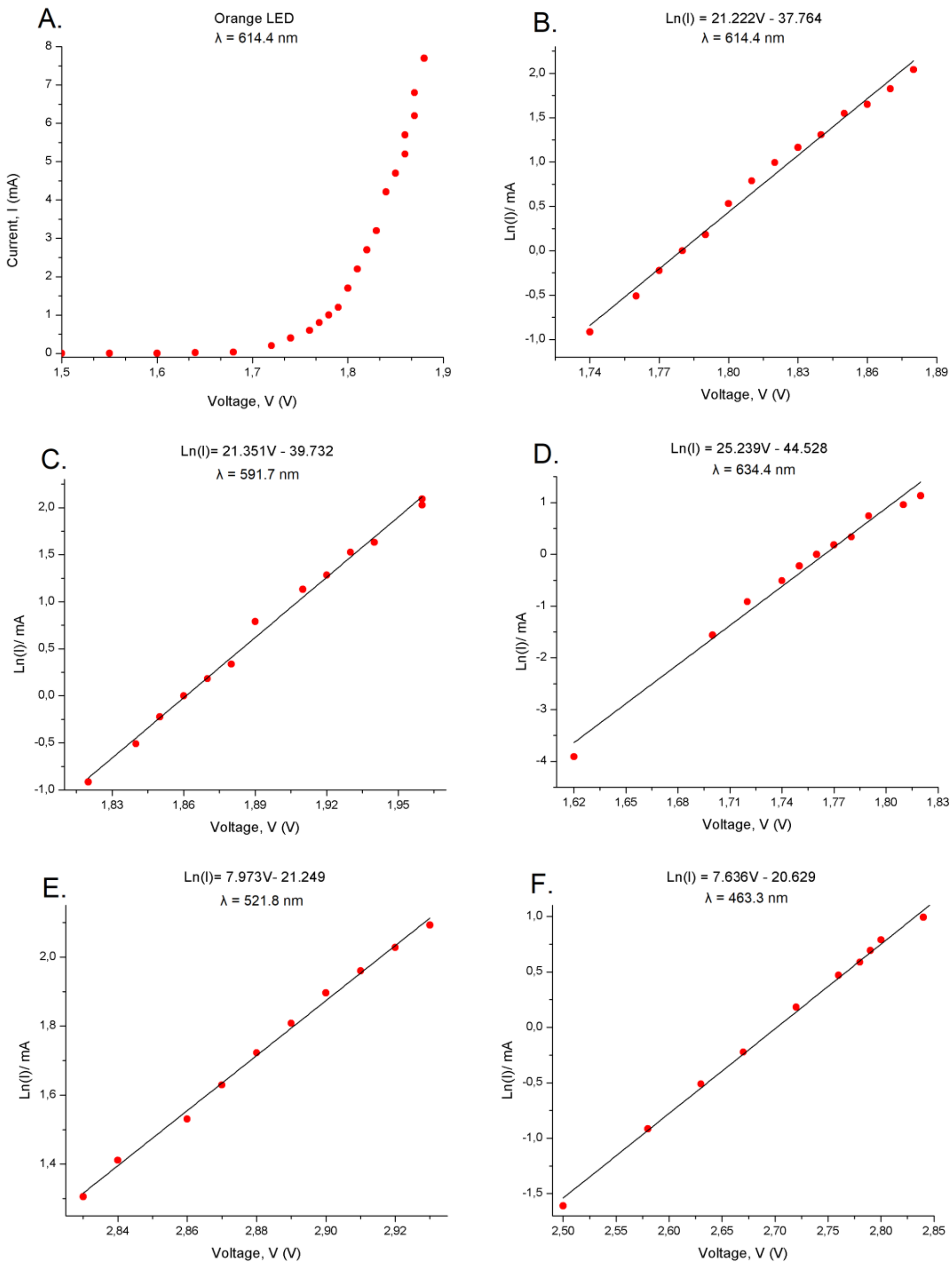

Fig.2. (A): Typical curve current (I) versus voltage (V) for an orange LED built from data as obtained with the Arduino Uno cart circuit shown in Fig.1; (B) to (F): Linear part of the Ln(I) versus V for the orange, yellow, red, green, and blue LED, respectively. In all cases, the linear fitting equation is presented. The wavelength associated to each LED is indicated.

It is important to say the major source of uncertainties come from the Arduino card measurements. From the circuit shown in Figure 1, the electrical current across the LED, *I*, satisfies

$$I = \frac{V - V_{LED}}{R} \tag{6}$$

where V = 5 V, V_{LED} is the fall voltage in the LED, and R = 330 Ω having a tolerance of 5%. The sensitivity in the V measurement was 0.01 V, as programmed in the Arduino card. In this cases, the experimental uncertainty u(V) can be estimated as [15]

$$u(V) = \frac{0.01V}{2\sqrt{3}} \quad (7)$$

On the other hand, the uncertainty in *I, u(I)* can be derived from Equation (6) and the result is [15]

$$u(I) = \sqrt{\left(\left[-\frac{V-V_{LED}}{R^2}\right]u(R)\right)^2 + \left(\left[-\frac{1}{R}\right]u(V_{LED})\right)^2} \quad (8)$$

Having into account that $u(R)$ = 2x10^1 Ω, and from Equation (7), $u(V)$ = 0.003 V, $u(I)$ ranges from 1x10^{-2} mA and 3x10^{-2} mA. All the uncertainties were reported using only one significant digit as recommended by JCGM [16]. Because the value of $u(V)$, all the values of Vg in Table 1 have three decimal positions.

Fig, 4 shows the V_g versus ν plot. Full circles and the red line represent the experimental data and their linear fitting, respectively. The fitting equation is $V_g = (3.967x10^{-15})\ \nu$. The fitting was carried out using the function ***linest*** implemented in Excel by Microsoft application. This function, working with the unweighted minimum square method [15], calculates the slope and intercept in a linear fitting equation as well as their respective uncertainties. Because theoretically, V_g as a function of ν cross the origin, the intercept value was fixed in zero, hence the slope and its uncertainty were 3.976x10^{-15} V and 3x10^{-16} V, respectively. Both Planck´s constant and its uncertainty can be obtained by multiplying the former values by the absolute value of the electron charge. In this way, Planck´s constant measured was

$$h = (6.4 \pm 0.4)x10^{-34} Js \quad (9)$$

In Equation (9), the uncertainty was expressed using two significant figures; hence, the best estimate has only two decimal digits [16]. Calculating the percent uncertainty, the value is less than 6%.

TABLE 1. Linear fitting parameters for the Figs. 1 (B-F). Vg estimated values for the LEDs, and their respective wavelength and frequency emitted at maximum intensity.

Color	***m (V^{-1})***	***b***	***λ (nm)***	***ν (Hz)***	***V_g (V)***
Red	25.2	44.5	634.4	4.729E+14	1.754
Green	7.9	21.2	521.8	5.749E+14	2.665
Yellow	21.6	39.7	591.7	5.070E+14	1.861
Orange	21.2	37.8	614.4	4.883E+14	1.779
Blue	7.6	20.6	463.3	6.475E+14	2.702

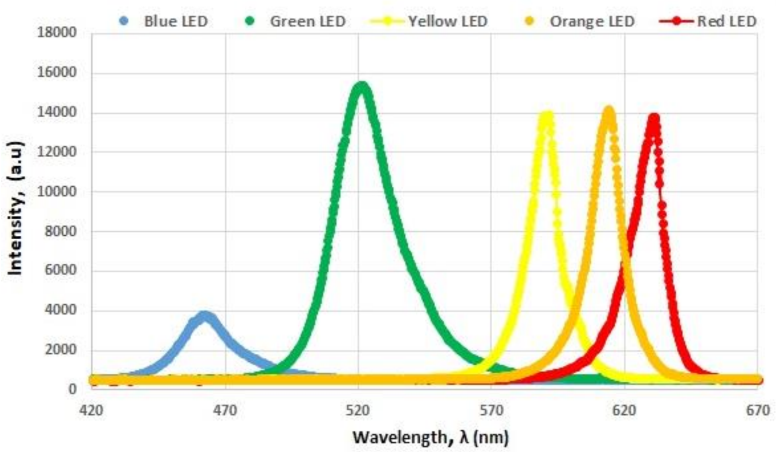

Fig .3. Emission spectra for all the LEDs studied in this work.

On the other hand, using the exact Planck´s constant exact value reported at NIST WEB [17], 6.626 070 15x10^{-34} Js, a discrepancy could be estimated, resulting in 0.256x10^{-34} Js, and a percent discrepancy of 4% approximately.

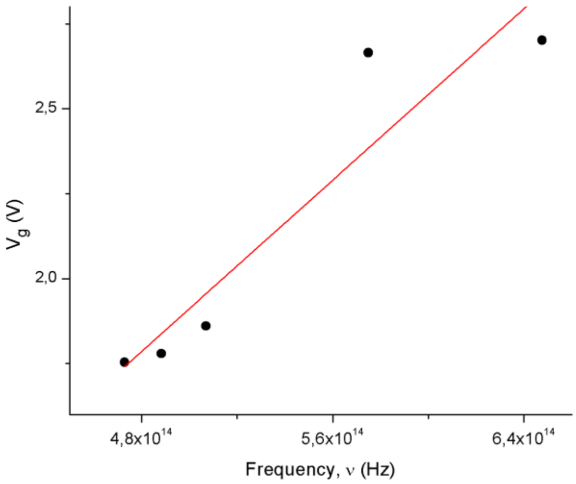

Fig. 4. Experimental data and the fitting line of the Vg as a function of ν values. The data were taken from the last two columns in Table 1.

Finally, to check if a linear fitting is acceptable in Fig. 4, the residual criteria were applied. As it is well known, in this context the residuals are calculated as the difference between the experimental and the respective fitted value. Irregular distribution of residuals is an indicative that the fitting is proper [15]. Fig. 5 shows the residuals, in which the disordered or random distribution of the points with respect to the zero value suggests the proposed linear fitting of V_g as a function of ν is reasonable. In this way, the procedure to measure Planck´s constant is suggested to be proper.

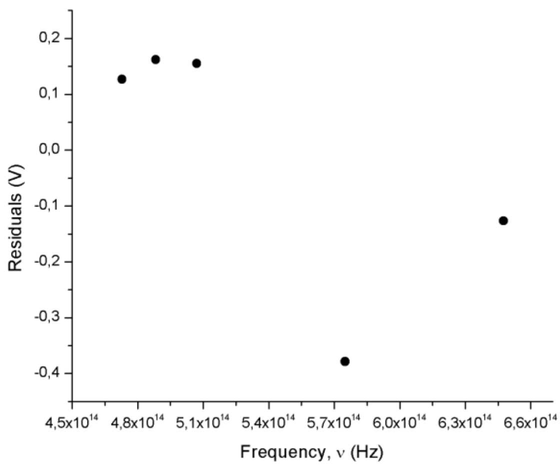

Fig. 5. Plot of residuals for the linear fitting of V_g versus ν. The linear fitting equation of the data in Fig. 4 is $V_g = (3.967x10^{-15})\ \nu$.

As a final comment, it can be said that a good number of semiconductors have been integrated into the use of a technological device as an Arduino cart simulating, programming, and using it as a measurement and a data acquisition system. In addition, metrological concepts and criteria must be studied and applied to report the results of the measurement method implemented. STEM strategies and skills are integrated into this work. Student motivation to study both fundamental and applied physics more deeply is expected from the implementation of this type of laboratory strategies.

3. Conclusions

A procedure for Planck´s constant determination using commercial light-emitting diodes was implemented. The procedure was designed to be carried out in an introductory university physics course dealing with modern physics concepts. The value obtained for the Planck´s constant was h = (6.4 ± 0.4) $x10^{-34}$ Js, with a percent uncertainty of less than 7%. Estimating the percent discrepancy of the measured value in comparison to the NIST exact one, the result is 4%. A STEM focus was used.

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