

Interpolation methods for polarimetric cameras

Métodos de interpolación para las cámaras polarimétricas

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ABSTRACT:

Polarimetric cameras enable the capture of physical scene properties and allow the observation of structures and contrasts that are invisible in conventional intensity-based imaging; therefore, they are increasingly useful in applications such as remote sensing, industrial inspection, biomedical imaging, and autonomous vision. They are Division of Focal Plane (DOFP) sensors that contain a linear polarizer in front of each pixel grouped in 2 by 2 orientations. In addition, color polarimetric cameras include color Bayer filters. These cameras provide a snapshot multi-channel intensities measurement but spatially separated and to have the full resolution, interpolation should be performed. We compare four interpolation methods: Bilinear, Cubic Spline, Modified Akima (Makima), and Fourier domain interpolation. To compare the different methods, simulated images with different frequency content are used. The study compares a monochrome 2×2 DoFP polarizer array (4 channels) and an RGB 4×4 color-polarization mosaic (12 channels). The results show that the local interpolators exhibit frequency and directions-dependent amplitude bias and edge overshoot, whereas Fourier-domain reconstruction is nearly exact for band-limited images, but ringing (Gibbs type behavior) appears near discontinuities due to violation of the band-limitation assumption. These error signatures provide practical guidance for selecting reconstruction methods in DoFP pipelines. MATLAB functions are provided, along with an example script to show how to use them.

Key words: DoFP polarimetry; Interpolation; Fourier-domain reconstruction; RGB color-polarization

RESUMEN:

Las cámaras polarimétricas permiten capturar propiedades físicas, revelar estructuras y alcanzar contrastes de imagen que no son observables mediante imágenes de intensidad convencional, lo que las hace especialmente relevantes en numerosas aplicaciones actuales. Estas cámaras incluyen sensores de polarización por división del plano focal (DOFP), presentando un polarizador lineal delante de cada píxel agrupado en orientaciones de 2x2. Además, las cámaras polarimétricas en color incluyen filtros Bayer de color. Estas cámaras proporcionan una medición instantánea de intensidades multicanal, pero espacialmente separadas y para obtener la resolución completa, se debe realizar algún tipo de interpolación. Comparamos cuatro métodos de interpolación: bilineal, spline cúbico, Akima modificado (Makima), e interpolación en el dominio de Fourier. Para comparar los diferentes métodos, se utilizan imágenes simuladas con diferente contenido en frecuencias. El estudio compara una matriz de polarizadores DOFP monocromática de 2×2 (4 canales) y un mosaico de color-polarización RGB de 4×4 (12 canales). Los resultados muestran que los interpoladores locales presentan un sesgo de amplitud dependiente de la frecuencia y la dirección, así como oscilaciones en los bordes. En cambio, la reconstrucción en el dominio de Fourier es casi exacta para imágenes con ancho de banda limitado, pero aparecen oscilaciones (comportamiento tipo Gibbs) cerca de las discontinuidades debido a que no se cumple la limitación de banda. Esto proporciona una guía práctica para seleccionar el método de reconstrucción dependiendo del tipo de imágenes esperado. Se dan las funciones en Matlab para realizar las interpolaciones, así como un script de ejemplo.

Palabras clave: Polarimetría por división del plano focal; Interpolación, Reconstrucción en el dominio de Fourier; Polarización en diferentes canales de color

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1. Introduction

Polarization imaging have many applications in diverse fields as remote sensing [1], astronomy [2], industry [3], bio photonics [4], machine vision [5] and general contrast enhancement [6]. There are several architectures for Polarimetric imaging system.

In the Division of Time (DoT) imaging polarimetry, polarization information is obtained sequentially either by rotating a wave plate [7], modulating the retardance of a liquid crystal panel [8], etc. The drawback of these methods is the acquisition time.



Division-of-amplitude (DoAm) [9] polarimetry is one of the well-established methods for rapid polarimetric measurement, allowing real-time retrieval of the Stokes vector. This approach eliminates temporal modulation by splitting the incoming beam into several optical paths, each of which is analyzed independently. Nevertheless, this approach generally relies on multiple optical channels and beam splitters, which increase the system's complexity, physical footprint, and alignment demands

Division of Aperture (DoAp) [10] systems partition the entrance aperture into multiple regions by laterally shifting mini-lens arrays within the optical path. Each region incorporates a different polarization analyzer, enabling the system to capture distinct polarization images on a single sensor.

Division of focal plane (DoFP) systems are especially attractive as they enable snapshot acquisition without moving parts, making them suitable for dynamic scenes and real-time application [7]. Hence, this work focuses on DoFP polarimetric cameras.

Division of focal plane (DoFP) polarimetric cameras enable snapshot acquisition of polarization-resolved information, making them highly attractive for applications such as remote sensing, advanced material inspection, biomedical imaging, and autonomous vision, where polarization information reveals scene properties beyond conventional intensity imaging. These cameras integrate micro-polarizer arrays at the sensor plane to provide snapshot of multi-channel intensity measurements. Note here that micro-polarizer arrays based DoFP sensors considered in this paper are incomplete Stokes systems because they measure only linear polarization components (S_0 , S_1 , S_2). Circular polarization information (S_3) can be obtained by adding modulation elements in front of these sensors. The mosaic architecture of polarized sensors spatially subsamples each analyser channel, and as a result, the total effective spatial resolution is reduced, and interpolation artifacts are introduced unless a dedicated reconstruction step is applied before computing polarization quantities.

Numerous demosaicking algorithms have been developed to recover full resolution polarization information from the raw mosaic data produced by DOFP sensors [11]. Despite these advanced methods, interpolation baselines remain widely used in practice and literature, serving as reference methods and as building blocks in larger reconstruction systems and processing pipelines [12-14].

Under this scenario, comparing different interpolation strategies is essential to assess their suitability for the reconstruction of polarization information in DoFP imaging systems. In this paper, the core spatial behaviour of four reconstruction interpolations that are frequently encountered in DoFP polarimetry is analysed: (i) bilinear interpolation [12], (ii) cubic-spline interpolation [12], (iii) modified Akima (Makima) interpolation [15], and (iv) Fourier-domain reconstruction via replica separation [16]. We present an interpretable benchmark reference that establishes a connection between reconstruction error signatures and spatial/frequency content such as discontinuities, bandwidth, and orientation rather than introducing a novel algorithm. We also compare the behaviour of two mosaic geometries that are frequently encountered in practice: a monochrome 2×2 DoFP polarizer mosaic and an RGB 4×4 DOFP colour polarization mosaic.

The outline of this paper is as follows. Section 2 provides a brief overview of the two common geometries used in polarimetric cameras with microgrid polarimeters. Linear, Makima, and cubic-spline interpolations can be performed using built-in MATLAB functions; therefore, in Section 3 we describe how to implement interpolation in the Fourier domain. We outline the basic Fourier-transform properties relevant to the interpolation process and explain the procedure for both monochrome and RGB polarimetric cameras. Links to the MATLAB functions and scripts used for all interpolation methods are also provided. Section 4 presents an error analysis based on simulated data for the different interpolation approaches, and Section 5 summarizes the main conclusions.

2. DOFP mosaic geometries and images sampling

In a DoFP monochrome polarized sensor, each 2×2 unit cell contains four orientations of the micro polarizers (e.g., 0° , 45° , 90° , 135°), see Fig. 1a. Each orientation forms a single subsampled image on a grid at resolution of one-half in each direction. In an RGB color-polarized sensor, a 4×4 -unit cell combines micro polarizers with RGGG color filter following a Bayer pattern, see Fig. 1b. The resulting mosaic produces 12 distinct sub-images (4 polarization states $\times$ 3 colors), where green is sampled twice.

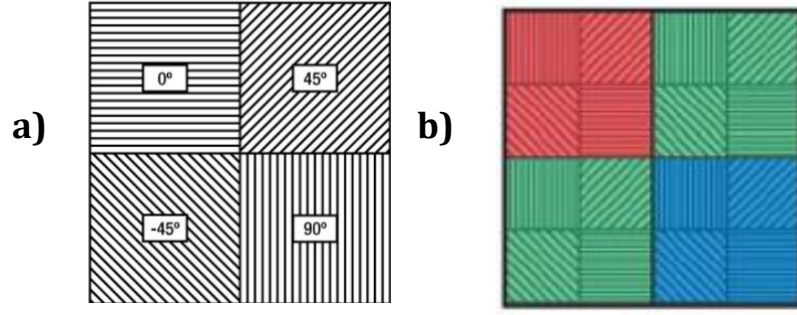


Fig.1. a) Monochrome polarized sensor unit cell 2x2, b) RGB polarized sensor unit cell 4x4

Given an input field, each polarization direction or each color-polarization direction is sampled at a different spatial coordinate. So, in each pixel we do not have the full color-polarization information. Then to evaluate the corresponding Stokes components on each pixel we need to perform some interpolation. In color polarization camera, there are spectral crosstalk among Bayer color channels due to the non-ideal spectral response of the color filter, so that the accuracy of the reconstruction is also influenced by these errors not only the interpolation methods. A method to correct this cross-talk can be found in [17]. The present study isolates the interpolation effect and assumes ideal color separation.

In this paper we will compare four interpolation methods with synthetic images that have different spatial frequency content. The quality of the reconstruction is assessed by comparing the reconstructed images with the known ground-truth simulated images.

3. Reconstruction methods

Spatial-domain interpolation methods are applied to the images captured by the polarimetric camera by using MATLAB's gridded interpolation function `interp2` [18]. Bilinear interpolation uses a piecewise planar model on the local neighborhood, cubic-spline interpolation uses a smooth cubic spline surface, and makima uses a modified Akima piecewise cubic Hermite formulation designed to reduce overshoot while preserving local shape [15]. For the RGB sensor, the two green instances are reconstructed and then averaged to produce a single green channel per polarization state. **MATLAB code is provided for all methods to facilitate their direct use on user data: [Monochrome](#) and [RGB](#).**

In this section we explain the Fourier Transform interpolation method that exploits the periodicity of the mosaic [16]. Both monochrome and color polarimetric sensor will be studied. Before introducing the method, we briefly summarize some properties of the Fourier Transform that will be useful to explain the reconstruction in the Fourier Domain.

3.a. Basic Properties of the Fourier Transform [19]

Let $G(\nu)$ be the Fourier Transform (FT) of $g(x)$

$$g(x) \xleftrightarrow[TF^{-1}]{TF} G(\nu) \quad (1)$$

The FT of a comb of deltas is another comb of deltas with the inverse period,

$$\sum_k \delta(x - kT) \xleftrightarrow[TF^{-1}]{TF} \frac{1}{|T|} \sum_m \delta\left(\nu - \frac{m}{T}\right) \quad (2)$$

The FT of a shifted function is the same of the original function with an added linear phase proportional to the shift.

$$g(x - x_0) \xleftrightarrow[TF^{-1}]{TF} G(\nu) e^{-i2\pi\nu x_0} \quad (3)$$

Then, the FT of a shifted comb of deltas is given in eq. (4). The delta function is different from zero when the argument is equal to zero. So, we can introduce the phase factor inside the summatory.

$$\begin{aligned} \sum_k \delta(x - x_0 - kT) &\xleftrightarrow[TF^{-1}]{TF} \left\{ \frac{1}{|T|} \sum_m \delta\left(\nu - \frac{m}{T}\right) \right\} e^{-i2\pi\nu x_0} \\ &= \frac{1}{|T|} \sum_m \delta\left\{ \left(\nu - \frac{m}{T}\right) e^{-i2\pi\frac{m}{T}x_0} \right\} \end{aligned} \quad (4)$$

The FT of the product of two functions is the convolution between their Fourier transforms. So, the FT of the product of a function with a comb of deltas is given by Eq (5). We have the repetition of the function FT ($G(\nu)$) by a distance of $1/T$. We will assume in the next sections that the function is band limited with a band-width smaller than the repetition rate. Under these assumptions the replicas will not overlap.

$$g(x) \square \sum_k \delta(x - kT) \xleftrightarrow[TF^{-1}]{TF} \frac{1}{|T|} G(\nu) \otimes \sum_m \delta\left(\nu - \frac{m}{T}\right) \quad (5)$$

By combining Eqs. (4) and (5) one can obtain the FT of the product of a function with a shifted comb of deltas.

$$g(x) \square \sum_k \delta(x - x_0 - kT) \xleftrightarrow[TF^{-1}]{TF} \frac{1}{|T|} G(\nu) \otimes \left\{ \sum_m \delta\left(\nu - \frac{m}{T}\right) e^{-i2\pi\frac{m}{T}x_0} \right\} \quad (6)$$

For the monochrome camera the shift is equal to the half of the comb period $x_0 = T/2$, we obtain

$$\begin{aligned} g(x) \square \sum_k \delta\left(x - \frac{T}{2} - kT\right) &\xleftrightarrow[TF^{-1}]{TF} \frac{1}{|T|} G(\nu) \otimes \left\{ \sum_m \delta\left(\nu - \frac{m}{T}\right) e^{-i\pi m} \right\} \\ &= \frac{1}{|T|} G(\nu) \otimes \left\{ \sum_m \delta\left(\nu - \frac{m}{T}\right) (-1)^m \right\} \end{aligned} \quad (7)$$

In the bidimensional case with shifts “k” are multiple of $T/2$: $(k_x, k_y)T/2$ with (k_x, k_y) equal to 0 or 1, we have

$$\begin{aligned} g(x, y) \square \sum_{p, q} \delta\left(x - k_x \frac{T}{2} - pT\right) \delta\left(x - k_y \frac{T}{2} - qT\right) &\xleftrightarrow[TF^{-1}]{TF} \\ \frac{1}{|T|} G(\mu, \nu) \otimes \left\{ \sum_{m, n} \delta\left(\mu - \frac{m}{T}\right) e^{-i\pi m k_x} \delta\left(\nu - \frac{n}{T}\right) e^{-i\pi n k_y} \right\} \\ &= \frac{1}{|T|} G(\nu) \otimes \left\{ \sum_{m, n} \delta\left(\mu - \frac{m}{T}\right) \delta\left(\nu - \frac{n}{T}\right) (-1)^{m k_x + n k_y} \right\} \end{aligned} \quad (8)$$

where q and p are integer numbers. For the color polarimetric camera the shifts are a multiple of a $1/4$ of the period T : $(k_x, k_y) \frac{T}{4}$ with (k_x, k_y) equal to 0, 1, 2, 3. Then the previous equation becomes:

$$\begin{aligned}
 g(x,y) \square \sum_{p,q} \delta\left(x - k_x \frac{T}{4} - pT\right) \delta\left(x - k_y \frac{T}{4} - qT\right) &\xleftrightarrow[TF^{-1}]{TF} \\
 \frac{1}{|T|} G(\mu, \nu) \otimes \left\{ \sum_{m,n} \delta\left(\mu - \frac{m}{T}\right) e^{-i\frac{\pi}{2}mk_x} \delta\left(\nu - \frac{n}{T}\right) e^{-i\frac{\pi}{2}nk_y} \right\} & \\
 = \frac{1}{|T|} G(\nu) \otimes \left\{ \sum_{m,n} \delta\left(\mu - \frac{m}{T}\right) \delta\left(\nu - \frac{n}{T}\right) e^{-i\frac{\pi}{2}(mk_x + nk_y)} \right\} &
 \end{aligned} \tag{9}$$

In the digital Fourier Transform we are dealing with sampled functions in both domains. Let us suppose that we have a sampled function in a matrix of (N×N) pixels with a sampling distance $(\Delta x, \Delta y)$. The minimum period that can be represented corresponds to the distance between two consecutive pixels. This period will produce the maximum frequency. The sampling distance in the FT will be $(\Delta \mu, \Delta \nu)$, then the relation between the sampling distances in both domains will be

$$\mu_{\max} = \frac{1}{2\Delta x} = \frac{N}{2} \Delta \mu \Rightarrow \Delta \mu \Delta x = \frac{1}{N} \tag{10}$$

as it is shown in Fig. 2.

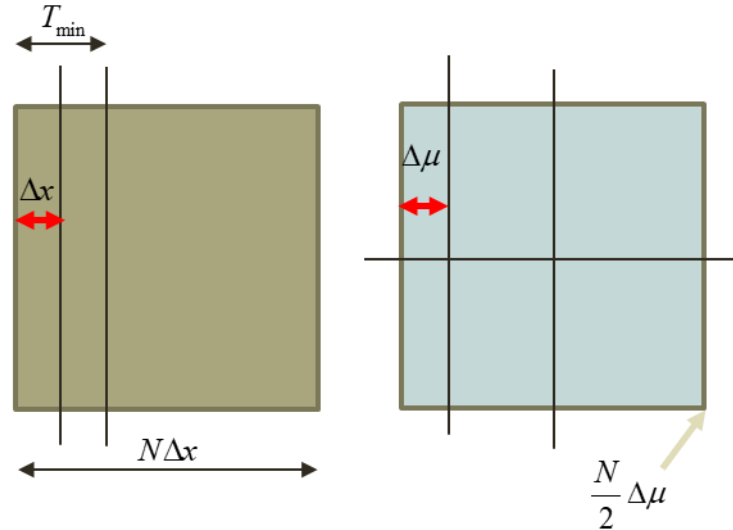


Fig.2. Relation between the sampling distances in spatial and Fourier domains

3.b. Monochrome Polarization Camera

In this case we have four different pixels with different polarization directions, as shown in Fig. 1a). So, given a beam with a spatial polarization distribution $S(x,y)$ each type of pixel will produce a different intensity distribution (I_k ; $k=1,...,4$) with an interlaced sampling. So, the intensity distribution captured by the sensor will be,

$$\begin{aligned}
 I(m,n) = & (I_1(x,y)) \sum_{n,m} \delta(x-2n\Delta x) \delta(y-2m\Delta y) \\
 & + (I_2(x,y)) \sum_{n,m} \delta(x-(2n+1)\Delta x) \delta(y-2m\Delta y) \\
 & + (I_3(x,y)) \sum_{n,m} \delta(x-2n\Delta x) \delta(y-(2m+1)\Delta y) \\
 & + (I_4(x,y)) \sum_{n,m} \delta(x-(2n+1)\Delta x) \delta(y-(2m+1)\Delta y)
 \end{aligned} \tag{11}$$

When performing the FT of this distribution it will appear different replicas of the FT of the intensity distributions given by each analyzer ($\tilde{I}_k$), each one affected with the corresponding sign due to the comb shifting as given by Eq (8). As we have commented before, the replicas will be shifted ($p N/2, q N/2$) pixels. Fig. 3 shows the location of the different replicas. The zero order (without shift) appear at the central part and is marked with 1. Note that the digital Fourier transform is periodical, for this reason the first replica in the X direction appear in zones marked with 2. The first replica in the Y direction is marked with 3, and so one.

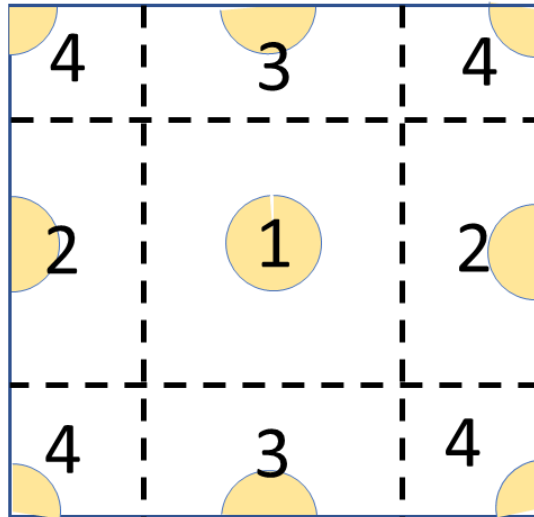


Fig.3. Location of each FT replica for a monochrome polarization camera. The circles indicate the zones of low frequencies

We consider that the first detector (A1) is centered, and the others are shifted. So, the Fourier transforms of the intensities corresponding to each detector will appear replicated with some sign changed. Fig. 4 shows the sign of each replica for the four different analyzers. The FT of the intensity distribution will be the addition of the four FT shown in Fig. 4.

$\tilde{I}_1$	$\tilde{I}_1$	$\tilde{I}_1$	$-\tilde{I}_2$	$\tilde{I}_2$	$-\tilde{I}_2$
$\tilde{I}_1$	$\tilde{I}_1$	$\tilde{I}_1$	$-\tilde{I}_2$	$\tilde{I}_2$	$-\tilde{I}_2$
$\tilde{I}_1$	$\tilde{I}_1$	$\tilde{I}_1$	$-\tilde{I}_2$	$\tilde{I}_2$	$-\tilde{I}_2$
$-\tilde{I}_3$	$-\tilde{I}_3$	$-\tilde{I}_3$	$\tilde{I}_4$	$-\tilde{I}_4$	$\tilde{I}_4$
$\tilde{I}_3$	$\tilde{I}_3$	$\tilde{I}_3$	$-\tilde{I}_4$	$\tilde{I}_4$	$-\tilde{I}_4$
$-\tilde{I}_3$	$-\tilde{I}_3$	$-\tilde{I}_3$	$\tilde{I}_4$	$-\tilde{I}_4$	$\tilde{I}_4$

Fig.4. Sign of each replica, corresponding to the FT of the intensity distribution corresponding to the four detectors.

Numerically, the distributions of each zone of Fig. 3 will be given by,

$$\left. \begin{aligned} \tilde{I}_{z1}(\mu, \nu) &= \tilde{I}_1(\mu, \nu) + \tilde{I}_2(\mu, \nu) + \tilde{I}_3(\mu, \nu) + \tilde{I}_4(\mu, \nu) \\ \tilde{I}_{z2}(\mu, \nu) &= \tilde{I}_1(\mu, \nu) - \tilde{I}_2(\mu, \nu) + \tilde{I}_3(\mu, \nu) - \tilde{I}_4(\mu, \nu) \\ \tilde{I}_{z3}(\mu, \nu) &= \tilde{I}_1(\mu, \nu) + \tilde{I}_2(\mu, \nu) - \tilde{I}_3(\mu, \nu) - \tilde{I}_4(\mu, \nu) \\ \tilde{I}_{z4}(\mu, \nu) &= \tilde{I}_1(\mu, \nu) - \tilde{I}_2(\mu, \nu) - \tilde{I}_3(\mu, \nu) + \tilde{I}_4(\mu, \nu) \end{aligned} \right\} \quad (12)$$

Now it is necessary to extract these four FT distributions and center them appropriately as it is shown in Fig. 5. In four matrices filled with zeros, we insert the corresponding different zones. For the $\tilde{I}_{z1}(\mu, \nu)$ only it is necessary to move the central zone (1) to the central zone of a zero filled matrix. For the $\tilde{I}_{z2}(\mu, \nu)$ zones (2a) and (2b) must be moved to the central part (1) of a zero filled matrix. For the $\tilde{I}_{z3}(\mu, \nu)$ zones (3a) and (3b) must be moved to the central part (1) of a zero filled matrix. Finally, For the $\tilde{I}_{z4}(\mu, \nu)$ zones (4a), (4b), (4c) and (4d) must be moved to the central part (1) of a zero filled matrix. After this reorganization, the inverse Fourier transform of each distribution is obtained

$$\left. \begin{aligned} I_{z1}(x, y) &= I_1(x, y) + I_2(x, y) + I_3(x, y) + I_4(x, y) \\ I_{z2}(x, y) &= I_1(x, y) - I_2(x, y) + I_3(x, y) - I_4(x, y) \\ I_{z3}(x, y) &= I_1(x, y) + I_2(x, y) - I_3(x, y) - I_4(x, y) \\ I_{z4}(x, y) &= I_1(x, y) - I_2(x, y) - I_3(x, y) + I_4(x, y) \end{aligned} \right\} \quad (13)$$

This can be written in matrix form as,

$$\begin{pmatrix} I_{z1} \\ I_{z2} \\ I_{z3} \\ I_{z4} \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{pmatrix} \Rightarrow I_z = MI \quad (14)$$

By inverting this equation, we obtain the interpolated intensity distributions due to the four detectors.

$$I = M^{-1}I_z \quad \text{where} \quad M^{-1} = 0.25 \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix} \quad (15)$$

3.c. Color Polarization Camera

In his case, we have four polarization orientation for each of the color filters, so twelve different sampling functions. The period, T, in this case is four pixels, then, the Fourier transform of the sampled function will contain replicas each one shifted $(mN/4, nN/4)$ pixels, and each one affected by a phase corresponding to the relative shift of the sensor as shown in equation (9).

The sampling with a polarization color sensor results in twelve different intensity distribution ($I_{p,c}$ $p=1,...,4$; $c=1,2,3$ (R,G,B)) with an interlaced sampling. Here “p” denotes polarizer orientation, and “c” the color channel. So, the intensity distribution captured by the sensor will be:

$$\begin{aligned}
 I(m, n) = & (I_{1,1}) \sum_{n,m} \delta(x - 2n\Delta x) \delta(y - 2m\Delta y) \\
 & + (I_{2,1}) \sum_{n,m} \delta(x - (2n+1)\Delta x) \delta(y - 2m\Delta y) \\
 & + (I_{3,1}) \sum_{n,m} \delta(x - 2n\Delta x) \delta(y - (2m+1)\Delta y) \\
 & + (I_{4,1}) \sum_{n,m} \delta(x - (2n+1)\Delta x) \delta(y - (2m+1)\Delta y) \\
 & + (I_{1,2}) \sum_{n,m} \delta(x - (2n+2)\Delta x) \delta(y - 2m\Delta y) \\
 & + (I_{2,2}) \sum_{n,m} \delta(x - (2n+3)\Delta x) \delta(y - 2m\Delta y) \\
 & + (I_{3,2}) \sum_{n,m} \delta(x - (2n+2)\Delta x) \delta(y - (2m+1)\Delta y) \\
 & + (I_{4,2}) \sum_{n,m} \delta(x - (2n+3)\Delta x) \delta(y - (2m+1)\Delta y) \\
 & + (I_{1,2}) \sum_{n,m} \delta(x - 2n\Delta x) \delta(y - (2m+2)\Delta y) \\
 & + (I_{2,2}) \sum_{n,m} \delta(x - (2n+1)\Delta x) \delta(y - (2m+2)\Delta y) \\
 & + (I_{3,2}) \sum_{n,m} \delta(x - 2n\Delta x) \delta(y - (2m+3)\Delta y) \\
 & + (I_{4,2}) \sum_{n,m} \delta(x - (2n+1)\Delta x) \delta(y - (2m+3)\Delta y) \\
 & + (I_{1,3}) \sum_{n,m} \delta(x - (2n+2)\Delta x) \delta(y - (2m+2)\Delta y) \\
 & + (I_{2,3}) \sum_{n,m} \delta(x - (2n+3)\Delta x) \delta(y - (2m+2)\Delta y) \\
 & + (I_{3,3}) \sum_{n,m} \delta(x - (2n+2)\Delta x) \delta(y - (2m+3)\Delta y) \\
 & + (I_{4,3}) \sum_{n,m} \delta(x - (2n+3)\Delta x) \delta(y - (2m+3)\Delta y)
 \end{aligned} \tag{6}$$

Note that the intensity distributions $I_{p,2}$ $p=1,\dots,4$ appear twice because the green sensor is repeated in different location as shown in Fig. 1b).

The Fourier Transform of the CCD camera intensity distribution is shown in Fig. 5. The FT in each diffraction order will be a linear combination of the FT of each of the 12 sampled functions,

$$\tilde{I}_\alpha = M \tilde{I}_{pc} \tag{17}$$

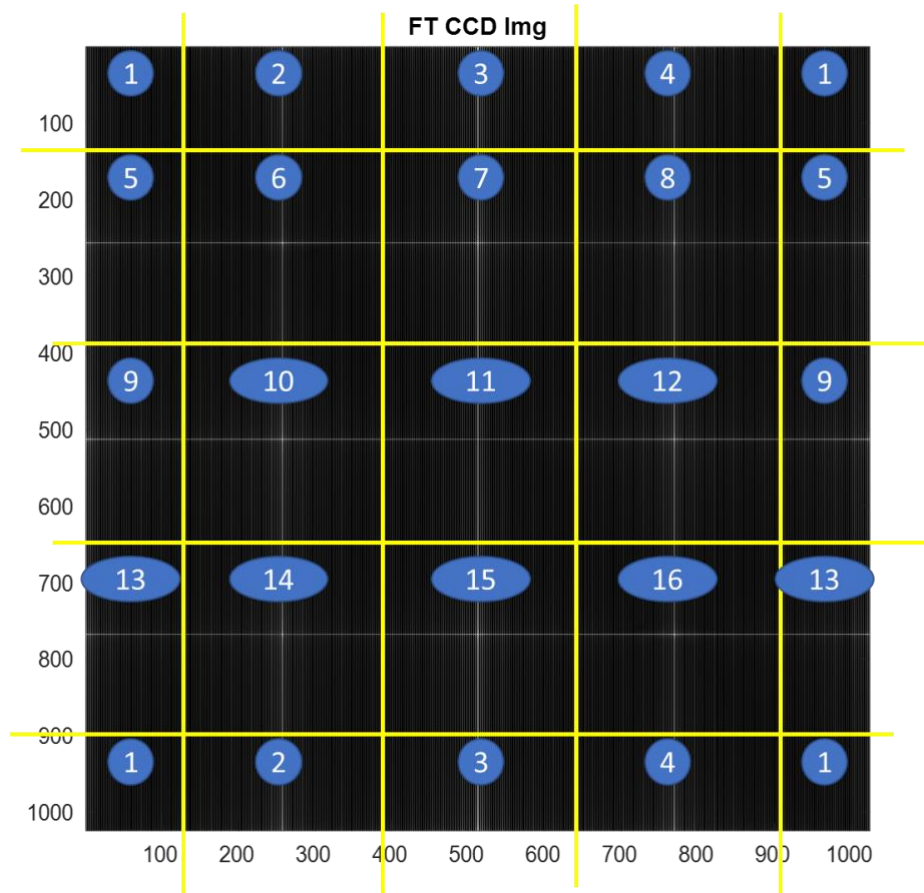


Fig.5. Fourier Transform of the CCD camera intensity distribution. The different diffraction orders appear numbered, being the marked with 11 the central one corresponding to (m=0, n=0).

$$\begin{pmatrix} \tilde{I}_1 \\ \tilde{I}_2 \\ \tilde{I}_3 \\ \tilde{I}_4 \\ \tilde{I}_5 \\ \tilde{I}_6 \\ \tilde{I}_7 \\ \tilde{I}_8 \\ \tilde{I}_9 \\ \tilde{I}_{10} \\ \tilde{I}_{11} \\ \tilde{I}_{12} \\ \tilde{I}_{13} \\ \tilde{I}_{14} \\ \tilde{I}_{15} \\ \tilde{I}_{16} \end{pmatrix} = \begin{pmatrix} 1 & -1 & -1 & 1 & 2 & -2 & -2 & 2 & 1 & -1 & -1 & 1 \\ 1 & i & -1 & -i & 0 & 0 & 0 & 0 & -1 & -i & 1 & i \\ 1 & 1 & -1 & -1 & 2 & 2 & -2 & -2 & 1 & 1 & -1 & -1 \\ 1 & -i & -1 & i & 0 & 0 & 0 & 0 & -1 & i & 1 & -i \\ 1 & -1 & i & -i & 0 & 0 & 0 & 0 & -1 & 1 & -i & i \\ 1 & i & i & -1 & -2 & -2i & -2i & 2 & 1 & i & i & -1 \\ 1 & 1 & i & i & 0 & 0 & 0 & 0 & -1 & -1 & -i & -i \\ 1 & -i & i & 1 & -2 & 2i & -2i & -2 & 1 & -i & i & 1 \\ 1 & -1 & 1 & -1 & 2 & -2 & 2 & -2 & 1 & -1 & 1 & -1 \\ 1 & i & 1 & i & 0 & 0 & 0 & 0 & -1 & -i & -1 & -i \\ 1 & 1 & 1 & 1 & 2 & 2 & 2 & 2 & 1 & i & 1 & 1 \\ 1 & -i & 1 & 1i & 0 & 0 & 0 & 0 & -1 & i & -1 & i \\ 1 & -1 & -i & i & 0 & 0 & 0 & 0 & -1 & 1 & i & -i \\ 1 & i & -i & i & -2 & -2i & 2i & -2 & 1 & i & -i & 1 \\ 1 & 1 & -i & -i & 0 & 0 & 0 & 0 & -1 & -1 & i & i \\ 1 & -i & -i & -1 & -2 & 2i & 2i & 2 & 1 & -i & -i & -i \end{pmatrix} \begin{pmatrix} \tilde{I}_{1,1} \\ \tilde{I}_{2,1} \\ \tilde{I}_{3,1} \\ \tilde{I}_{4,1} \\ \tilde{I}_{1,2} \\ \tilde{I}_{2,2} \\ \tilde{I}_{3,2} \\ \tilde{I}_{4,2} \\ \tilde{I}_{1,3} \\ \tilde{I}_{2,3} \\ \tilde{I}_{3,3} \\ \tilde{I}_{4,3} \end{pmatrix} \quad (18)$$

In a similar way as in the monochrome camera, each of the diffraction orders is isolated and centered in a zero filled matrix, then we inverse the equation (16),

$$\tilde{I}_{pc} = (M' M)^{-1} M' \tilde{I}_{\alpha} \quad (19)$$

Finally, the 12 intensity distributions are obtained by inverse Fourier transforming each of the $\tilde{I}_{pc}$ distributions.

$$I_{pc} = FT^{-1}(\tilde{I}_{pc}) \quad (20)$$

4. Simulated images and evaluation protocol

The final conclusions are similar for the monochrome and for the color polarimetric cameras. Therefore, to avoid unnecessary extending the exposition we only present the results corresponding to the color-polarimetric camera. For that, we have generated 12 functions with different spatial frequency contents each one in a matrix with 1024x1024 pixels. The following functions are generated: (1) Five sinusoidal functions with different orientations (these are band limited functions); (2) Two Gaussians with 50- and 100-pixels width that are almost band limited functions; (3) A correlated random noise with a narrow spectral band, and (4) four rectangular functions that are not band limited functions. Each one of these functions are sampled following the scheme of polarization-color given in Fig. 1 and Fig 6 (Right) to simulate the image captured by the color polarization camera. A portion of this image is shown in Fig. 6 (Left).

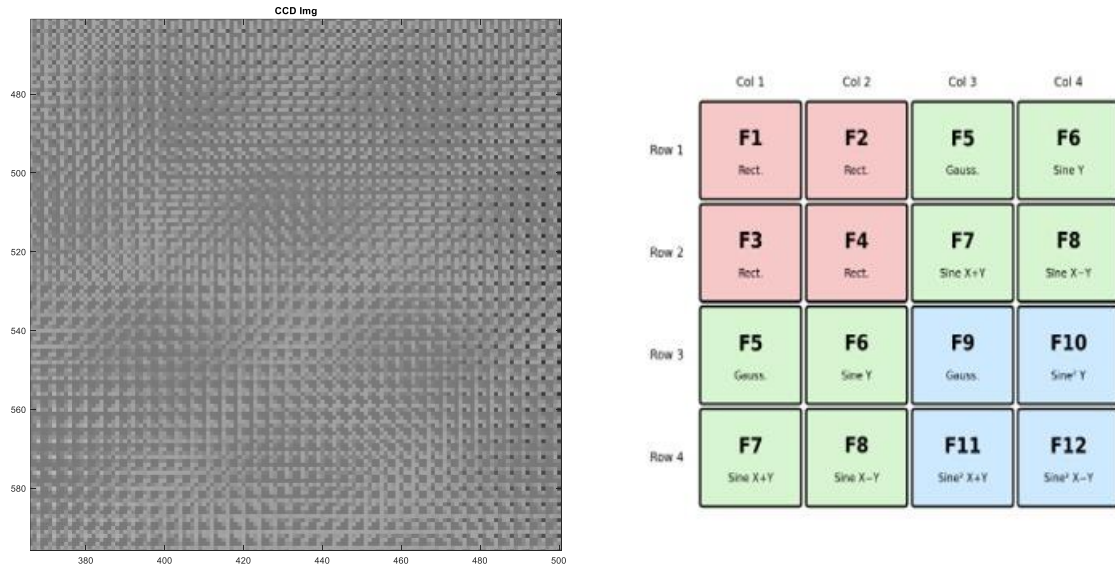


Fig.6. (Left) Enlarged portion of the simulated image captured by the polarization color camera. 12 independent 1024x1024 test sampled functions were generated and each pixel of the camera is taken from one of the 12 functions (Fn) according to the scheme on the right corresponding to the 12 color-polarization channels.

In Figs. 7-10 we show a summary of the different interpolation methods performances. Each row corresponds to a different function with different spatial frequency bandwidth. The left column shows the normalized reconstructed function to the range [0,1], the central column represents the error, i.e., the difference between the original function and the reconstructed one, and the right column shows a histogram of the corresponding error. The red circles and red numbers correspond to numerical error values to illustrate the magnitude of the error at some selected pixels. To improve the visibility we have written the values of some of them outside the figure

In Figs. 7–10(a), the results corresponding to one of the sinusoidal functions are shown. This function is band-limited, with a spatial frequency that lies within the central diffraction order. By comparing the different methods, we observe that the Fourier method performs best, followed by cubic spline, Makima, and linear interpolation. In this case, the original function is recovered almost perfectly with the Fourier transform method; the errors are on the order of 10^{-7} , corresponding to numerical precision, as can be seen in the central and right columns.

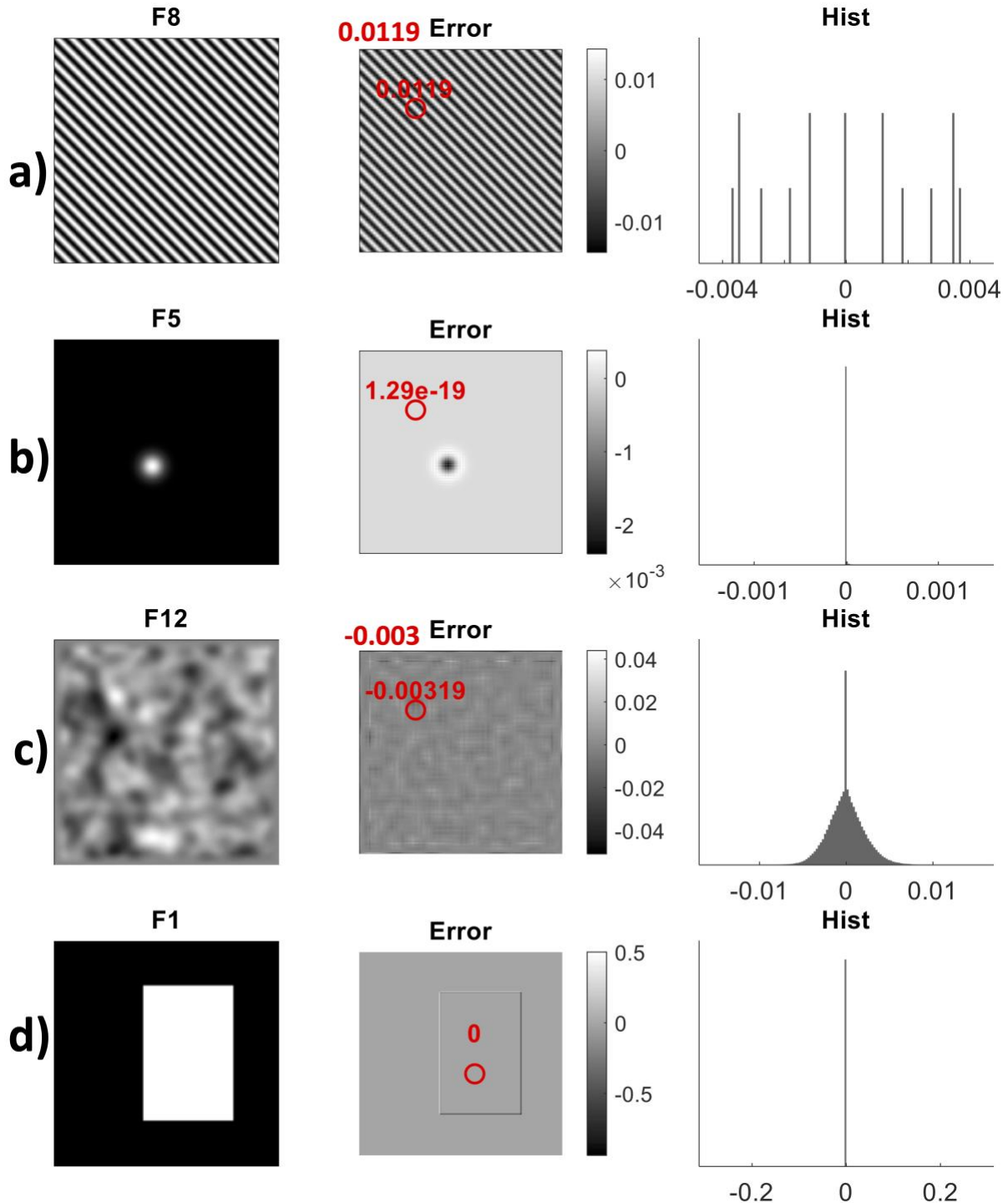


Fig.7. LINEAR INTERPOLATION. Rows correspond to some of the different functions results: (a) sinusoidal, (b) Gaussian, (c) correlated random noise and (d) rectangular function. The Left column corresponds to the reconstructed function after the interpolation with values between $[0,1]$. The central column is the error (difference between the original function and the interpolated one). The right column is a histogram of the corresponding error.

In Figs. 7–10(b), the results corresponding to the Gaussian function are shown. Since this function is nearly band-limited, with its bandwidth contained within a single replica, the errors in the Fourier method remain negligible. The errors obtained with the other methods follow the same trend as in the sinusoidal case. However, in this scenario the errors are smaller because the bandwidth of the Gaussian is narrower than that of the sinusoidal function.

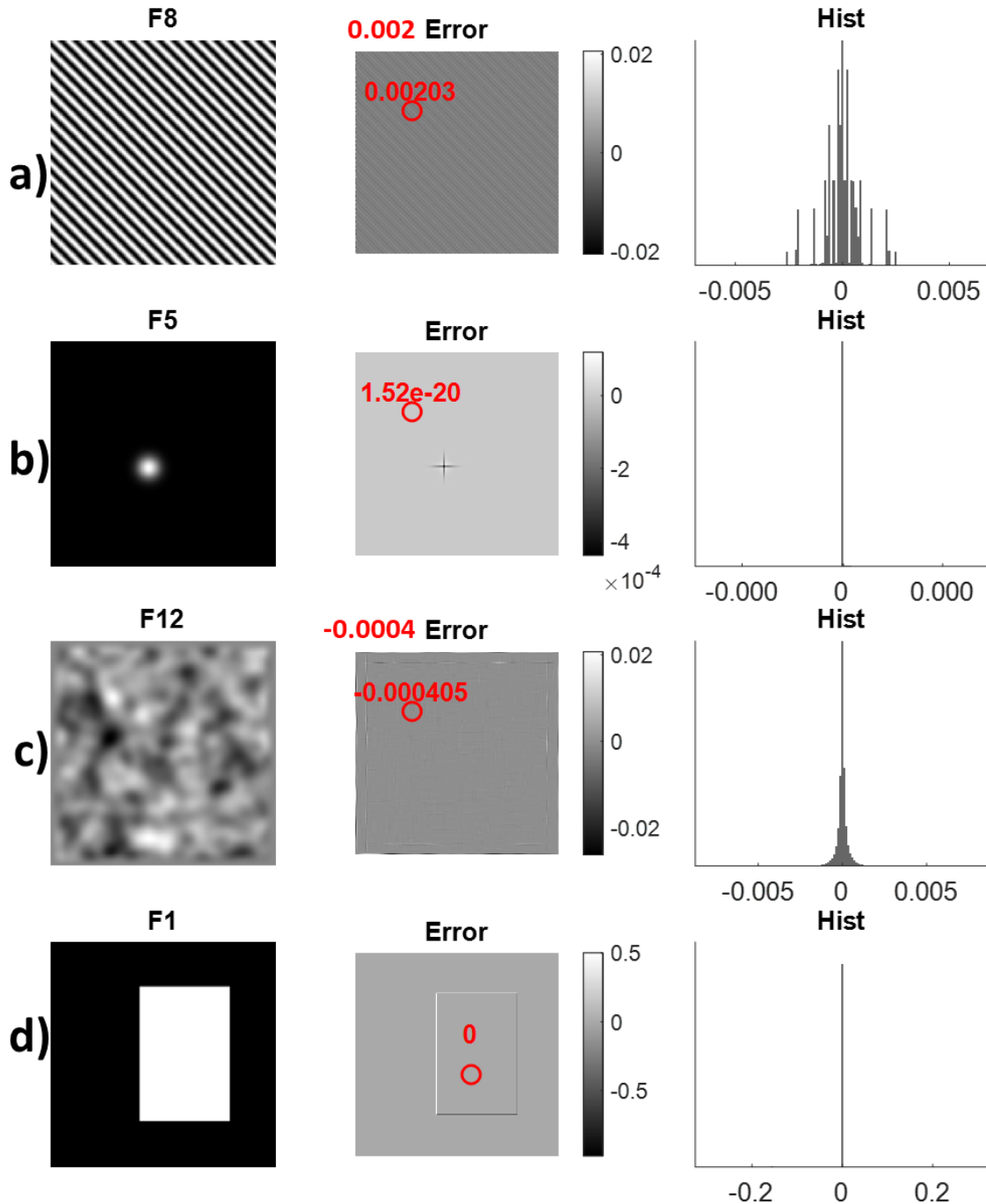


Fig.8. MAKIMA INTERPOLATION. Rows correspond to some of the different functions results: (a) sinusoidal, (b) Gaussian, (c) correlated random noise and (d) rectangular function. The Left column corresponds to the reconstructed function after the interpolation with values between [0,1]. The central column is the error (difference between the original function and the interpolated one). The right column is a histogram of the corresponding error.

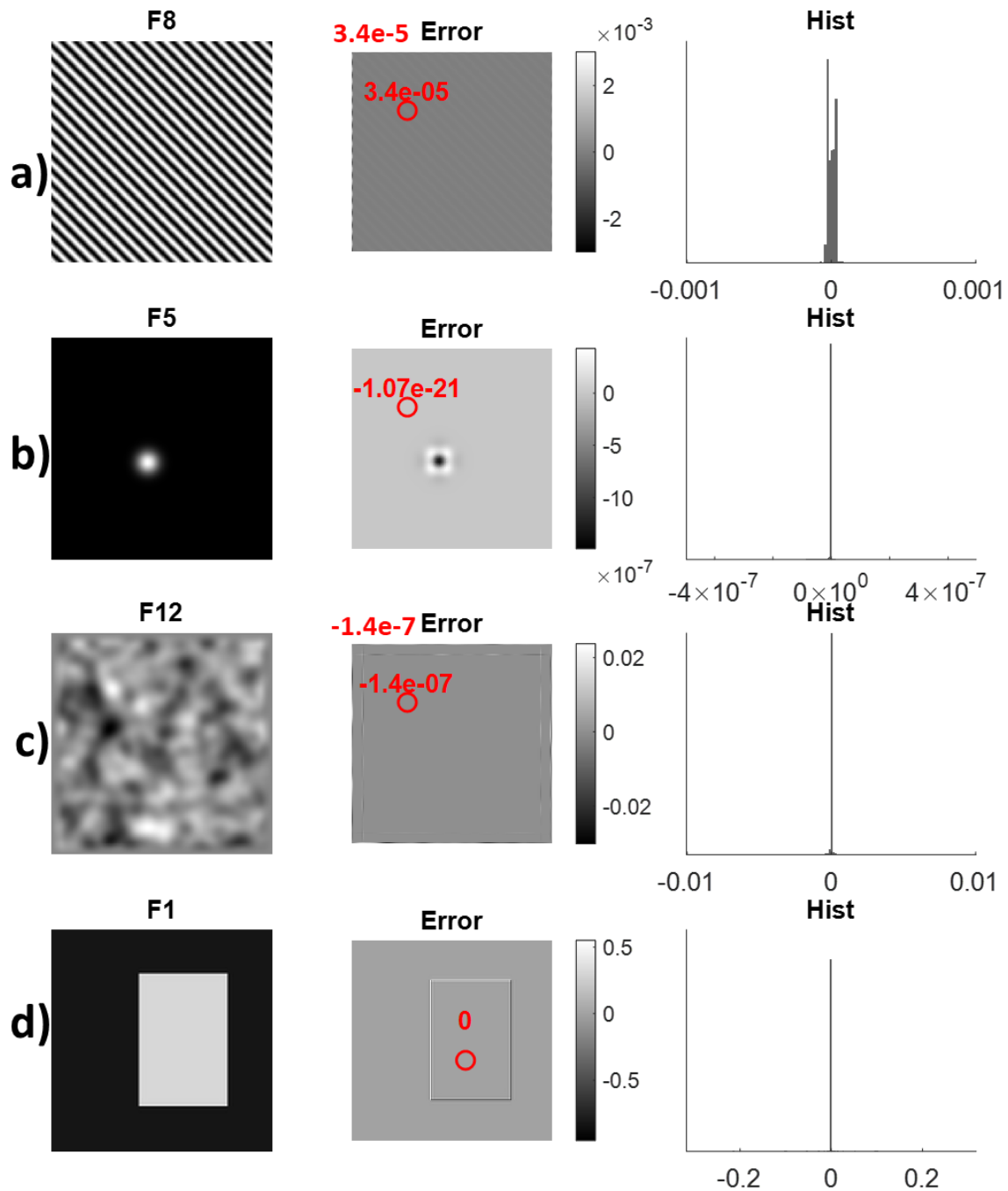


Fig.9. CUBIC SPLINES INTERPOLATION. Rows correspond to some of the different functions results: (a) sinusoidal, (b) Gaussian, (c) correlated random noise and (d) rectangular function. The Left column corresponds to the reconstructed function after the interpolation with values between [0,1]. The central column is the error (difference between the original function and the interpolated one). The right column is a histogram of the corresponding error.

Figs. 7–10(c) correspond to the case of a random correlated signal whose bandwidth also lies within a single Fourier-transform replica (see left column in Figs. 7–10(c)). In this case, the error range is noticeably larger (central and right columns). It is important to remark that Fourier-transform algorithms inherently assume periodicity of the input function—that is, the matrix shown in the left column is implicitly repeated periodically. As a consequence, discontinuities appear along the four image borders, generating

high-frequency components that lead to interpolation errors in those pixel lines. This effect becomes evident when selecting any pixel away from the borders, such as the one highlighted in Fig. 7(c), central column, where the associated error is very small (10^{-5}). Therefore, in this example the largest errors are concentrated at the image edges due to the discontinuity.

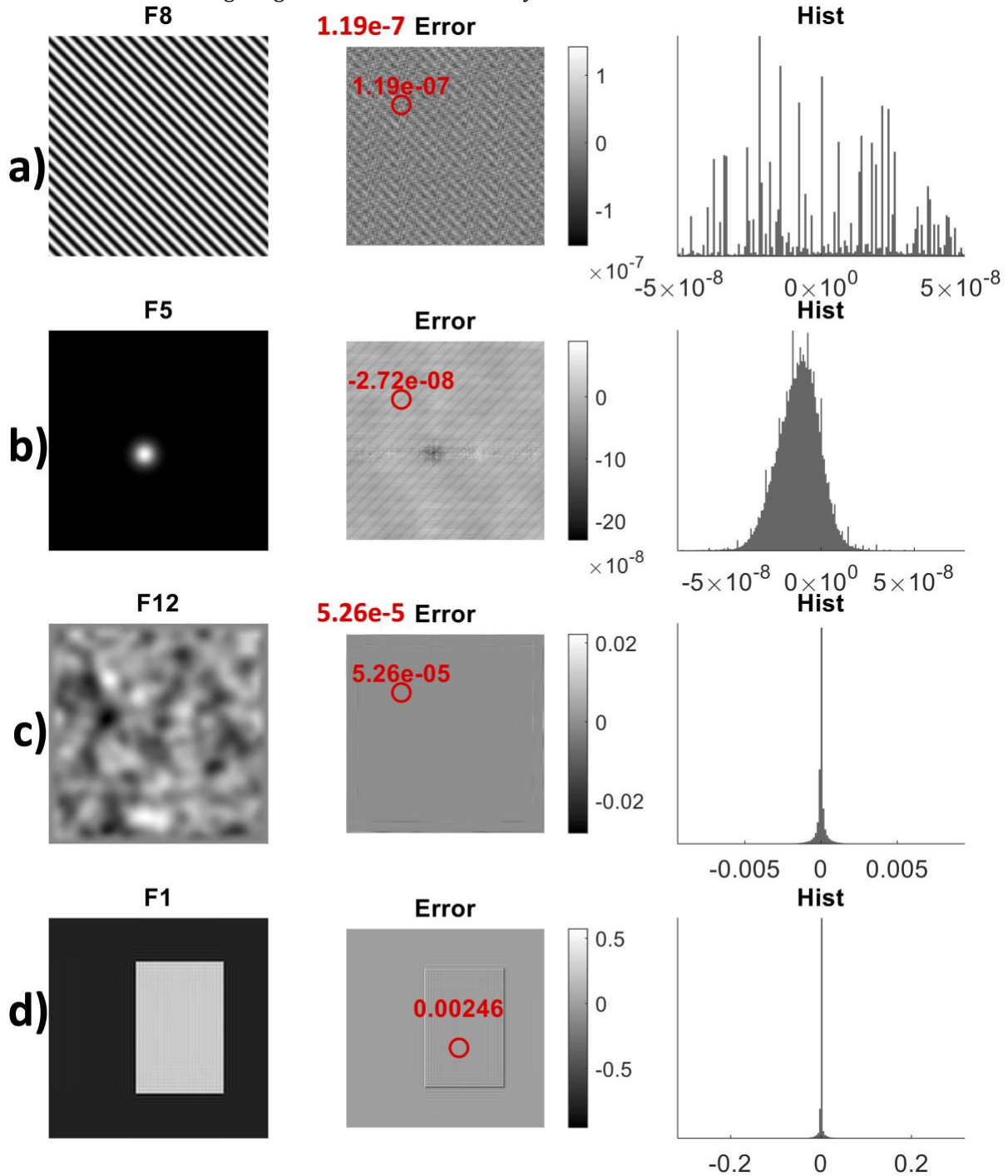


Fig.10. FOURIER TRANSFORM INTERPOLATION. Rows correspond to some of the different functions results: (a) sinusoidal, (b) Gaussian, (c) correlated random noise and (d) rectangular function. The Left column corresponds to the reconstructed function after the interpolation. The central column is the error (difference between the original function and the interpolated one). The right column is a histogram of the corresponding error.

Finally, Figs. 7–10(d) correspond to the rectangle function. Due to the discontinuities at the rectangle edges, this function is not band-limited, and the resulting errors are therefore concentrated along its borders. All

interpolation methods yield nearly the same error distribution. Away from the edge regions, the error becomes almost negligible

To examine in detail how the error evolves with the frequency content of the signal, Figs. 11 and 12 present the root-mean-square error (RMSE) on a linear scale (right) and the maximum absolute error (MaxAbs) on a logarithmic scale (left). In Fig. 11, we simulate a Gaussian signal with different widths (W). As W increases, the frequency bandwidth decreases, and all interpolation methods exhibit lower errors. The Fourier-domain reconstruction yields the smallest error, followed by cubic spline, Makima, and linear interpolation. In turn, Fig. 12 shows the case of a sinusoidal function with increasing spatial frequency. As expected, all spatial-domain interpolation methods degrade as the sinusoid approaches the Nyquist limit (for the monochrome sensor, the Nyquist limit is 0.25 cycles/pixel), whereas the Fourier method maintains an error below 10^{-8} across the entire bandwidth.

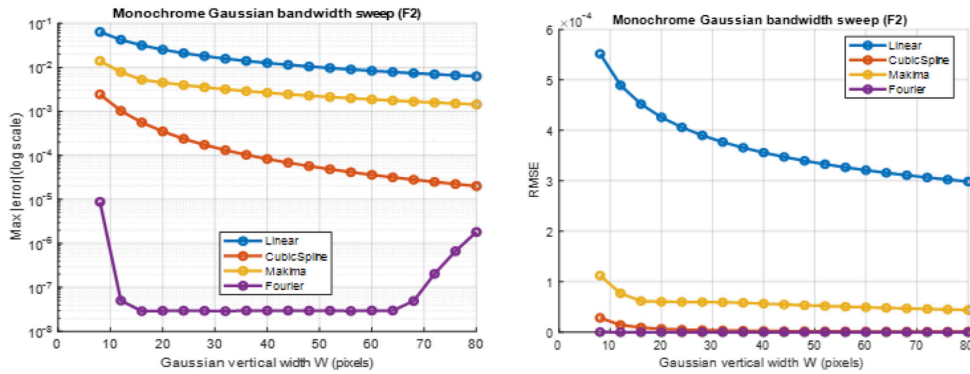


Fig.11. Monochrome Gaussian width sweep, Right: RMSE with linear scale vs W , Left: MaxAbs with log vs W Monochrome Gaussian width sweep—error metric versus W for each method.

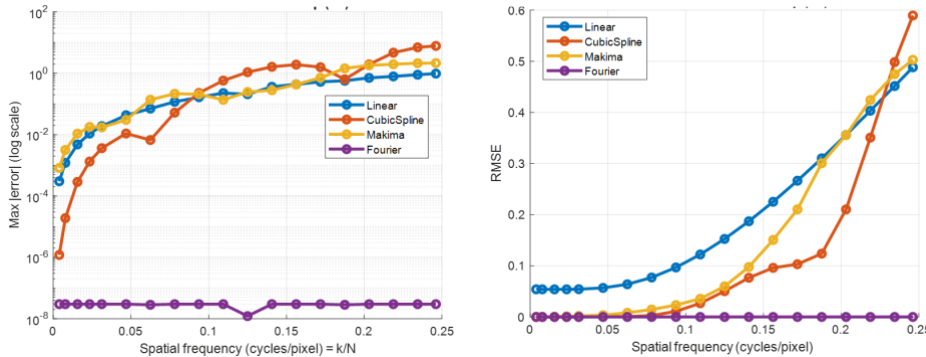


Fig.12. Monochrome sinusoid spatial-frequency sweep for channel F3 using DFT-coherent frequencies $f=k/N$ (sub-lattice Nyquist at 0.25 cycles/pixel). (a) Maximum absolute error (log scale). (b) RMSE (linear scale).

5. Conclusion

In this paper, we have presented a comparative analysis of several widely used interpolation methods for Division of Focal Plane (DoFP) polarimetric cameras, focusing on their fundamental spatial and frequency-domain behaviour. Specifically, we have evaluated bilinear, cubic-spline, Modified Akima (Makima), and Fourier-domain reconstruction methods using controlled simulated images with varying frequency content, discontinuities, and orientations.

The results show that local spatial-domain interpolation methods exhibit characteristic error signatures, including frequency- and direction-dependent amplitude bias and edge overshoot. Among them, cubic-spline and Makima interpolation generally outperform bilinear interpolation, although they remain

sensitive to high spatial frequencies and sharp transitions. These limitations become increasingly evident as the signal bandwidth approaches the Nyquist limit of the subsampled polarization channels.

In contrast, the Fourier-domain reconstruction method provides nearly exact recovery when the underlying image content is band-limited. Under this condition, the reconstruction errors are reduced to numerical precision. However, when the band-limitation assumption is violated—such as near discontinuities or image borders—ringing artifacts appear. These effects are inherent to the Fourier-based approach and arise from the periodicity assumptions of the discrete Fourier transform.

Importantly, although the analysis has been presented mainly for the color DoFP configuration, similar conclusions hold for monochrome polarimetric sensors. The observed relationships between reconstruction errors and spatial-frequency content are therefore largely independent of the specific mosaic geometry.

Overall, this study does not aim to introduce a new demosaicking algorithm, but rather to provide an interpretable benchmark that links interpolation artifacts to image characteristics. The resulting insights offer practical guidance for selecting interpolation strategies in DoFP polarimetric pipelines, depending on the expected scene content and reconstruction requirements. To facilitate practical adoption, MATLAB implementations of all evaluated methods are provided, enabling readers to directly apply and assess the interpolations on their own data.

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