

Light–Matter Interaction in Quantum Emitters: An Introduction Using QuTiP

Interacción Luz-Materia en Emisores Cuánticos: Una introducción usando QuTiP

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ABSTRACT:

Quantum emitters constitute one of the most fundamental platforms for studying quantum light–matter interactions and play a central role in emerging quantum technologies. Here, we present an introduction for undergraduate and graduate students to the interaction between light and two-level quantum emitters based on numerical simulations implemented with QuTiP, an open-source Python framework for modelling open quantum systems. This tutorial combines numerical implementations with intuitive physical explanations and concise analytical derivations. The provided codes can be directly implemented and readily adapted to systematically explore the effects of different physical parameters, helping students develop physical intuition about quantum optical phenomena. The topics covered include spontaneous emission, coherent laser driving, the Mollow triplet, Rabi oscillations, photon antibunching, the Purcell effect, and the concept of polariton, among others. This tutorial is therefore intended as an accessible introduction to quantum optics for undergraduate and graduate students, while also providing a practical starting point for the numerical modelling of quantum emitters and nanophotonic systems using QuTiP.

Key words: quantum optics, quantum emitters, QuTiP, numerical simulations, nanophotonics, open quantum systems, spontaneous emission, photon antibunching

RESUMEN:

Los emisores cuánticos constituyen una de las plataformas fundamentales para el estudio de las interacciones cuánticas luz–materia y desempeñan un papel central en tecnologías cuánticas. En este tutorial, se presenta una introducción orientada a estudiantes de grado y máster sobre la interacción entre la luz y emisores cuánticos de dos niveles basada en simulaciones numéricas implementadas con QuTiP, una plataforma de código abierto en Python para el modelado de sistemas cuánticos abiertos. Este tutorial combina códigos y simulaciones en QuTiP con explicaciones físicas intuitivas y derivaciones analíticas concisas. Los códigos proporcionados pueden implementarse directamente y adaptarse fácilmente para explorar de manera sistemática los efectos de distintos parámetros físicos, ayudando a los estudiantes a desarrollar una intuición física sobre distintos fenómenos en óptica cuántica. Entre los temas abordados se incluyen la emisión espontánea, la excitación coherente mediante un láser, el triplete de Mollow, oscilaciones de Rabi, la emisión de fotones individuales, el efecto Purcell, y el concepto de polaritón, entre otros. Por tanto, este tutorial está concebido como una introducción accesible a la óptica cuántica para estudiantes de grado y máster, al tiempo que proporciona un punto de partida práctico para el modelado numérico de emisores cuánticos y sistemas nanofotónicos utilizando QuTiP.

Palabras clave: óptica cuántica, emisores cuánticos, QuTiP, simulaciones numéricas, nanofotónica, sistemas cuánticos abiertos, emisión espontánea, emisión de fotones individuales

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1. Introduction

Quantum emitters are microscopic systems capable of absorbing and emitting electromagnetic radiation in discrete energy quanta, called photons. The optical response of quantum emitters is determined by a discrete energy-level structure. In many situations, the optical response of the emitter can be accurately described by considering only two relevant energy levels. Two-level quantum emitters constitute promising platforms for emerging quantum technologies, as they can operate as single-photon sources in a wide range of quantum communication and quantum computing protocols [1,2]. Consequently, substantial efforts have been devoted to their physical implementation over the last decades. Today, high-quality realizations exist in gas-phase systems, such as atoms and ions [3–6], as well as in solid-state platforms, including molecules embedded in solid matrices [7,8], artificial atoms such as quantum dots [9,10], color centers [11,12], and, more recently, defects in two-dimensional materials [13,14].

The interaction between light and a two-level quantum emitter represents one of the most elementary scenarios of quantum light–matter interaction. For this reason, these systems are widely used to introduce fundamental concepts in quantum optics, such as spontaneous emission and coherent (laser) driving. Despite their conceptual simplicity, their presentation in textbooks and tutorials often relies on extensive analytical derivations, which may hinder intuitive understanding, particularly at the undergraduate level.

Here, we introduce light–matter interactions in quantum emitters, with particular emphasis on the implementation of numerical simulations using QuTiP (specifically version 5.2.1), an open-source Python library for simulating the dynamics of open quantum systems [15,16]. Its high-level syntax enables focusing on the underlying physics rather than on low-level numerical implementation details. By implementing the codes provided in this tutorial and systematically varying the relevant physical parameters, undergraduate and graduate students can develop physical intuition about quantum light–matter interaction. Prior familiarity with Python programming is assumed, as well as a basic knowledge of quantum mechanics and Dirac notation.

In addition to the numerical framework, we present complementary analytical derivations that establish a clear connection between theoretical concepts and computational results. This combined approach makes the material particularly suitable for undergraduate and master’s-level courses in Quantum Optics and Nanophotonics. This work can also serve as a practical starting point for researchers interested in using QuTiP to model quantum emitters.

We begin in Section 2 by introducing the concept of a quantum emitter, together with the basic steps required to perform calculations in QuTiP. In Section 3, we address spontaneous emission, illustrating the phenomenon through numerical calculations based on the Lindblad master equation formalism. Section 4 is devoted to the interaction between a laser field and a quantum emitter. There, we describe numerical codes that enable the exploration of Rabi oscillations and emitter saturation. In Section 5, we present the quantum regression theorem and the calculation of the emission spectrum, which allows for analyzing the emergence of the Mollow triplet. In Section 6, we implement numerical procedures to compute the intensity correlation of the emitted light, introducing the concept of photon antibunching. Finally, we briefly examine the numerical modelling of a quantum emitter weakly coupled to an optical cavity mode within the Jaynes–Cummings model, introducing the Purcell effect, as well as the case of strong coupling, which allows us to introduce the concept of polariton.

2. Quantum objects in QuTiP

To model a two-level quantum emitter in QuTiP, we first represent its states and operators in a compact numerical form. To this end, QuTiP introduces a dedicated class to represent quantum systems, known as Qobj (Quantum Object). This class stores information about the matrix representation of a given operator or state. Consequently, quantum states, operators, Hamiltonians, and Liouvillian superoperators are all represented as instances of Qobj [15,16].

We begin by defining quantum states in QuTiP. With this aim, we consider a two-level quantum emitter, consisting of a ground state $|g\rangle$ (lower in energy) and an excited state $|e\rangle$ (higher in energy). These two states are separated by a well-defined energy difference $\hbar\omega_{eg}$ (see Fig. 1a), where $\hbar$ denotes the reduced Planck constant and ω_{eg} is the transition frequency. For example, $|g\rangle$ and $|e\rangle$ can represent electronic states

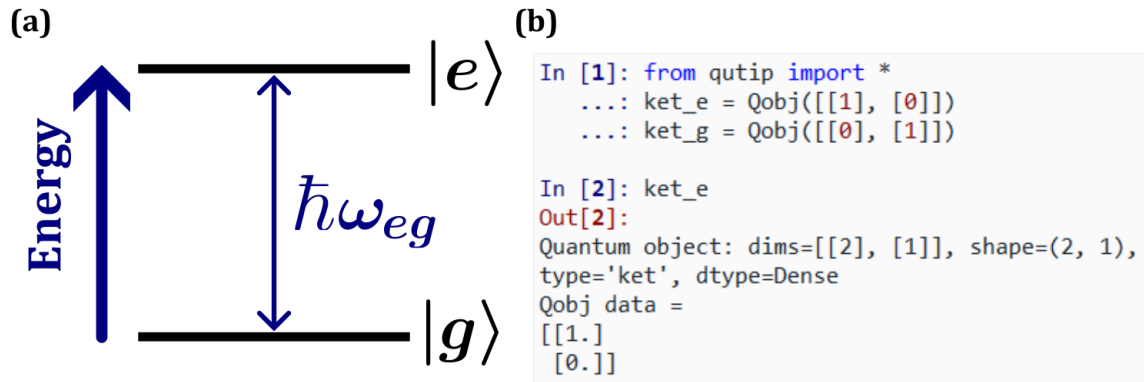


Fig. 1. Description of a two-level quantum emitter. (a) Schematic representation of a two-level quantum emitter with ground state $|g\rangle$ and excited state $|e\rangle$. The energy splitting between the two states is $\hbar\omega_{eg}$, with ω_{eg} known as the transition frequency. (b) Information stored in a Qobj in QuTiP (specifically, in ket_e defined in Code 2).

of an atom or a molecule [1].

A quantum state can be constructed as a Qobj in a manner analogous to the standard definition of matrices with the NumPy package in Python, shown in Code 1.

```
import numpy
a = numpy.matrix([1, 0]) # Horizontal (1x2) vector
a = numpy.matrix([[1], [0]]) # Vertical (2x1) vector
```

Code 1. Definition of horizontal and vertical matrix vectors using NumPy.

Just as NumPy can be used to define matrices in Python, QuTiP can be used to define quantum states and operators. For instance, the ground and excited states, represented as kets in Dirac notation, can be defined using QuTiP as described in Code 2.

```
from qutip import *
ket_e = Qobj([[1], [0]])
ket_g = Qobj([[0], [1]])
```

Code 2. Definition of the excited- and ground-state ket vectors using QuTiP.

Figure 1b shows the internal structure of a Qobj (concretely, ket_e), including its dimensions and numerical data. Specifically, the information stored in ket_e as a Qobj is its tensor structure (dims), its total matrix size (shape), its physical meaning (type), the numerical precision (dtype), and the matrix elements (data).

Further, the bra states $\langle e|$ and $\langle g|$ can be obtained by taking the Hermitian adjoint of the corresponding ket states using the `.dag()` function (Code 3).

```
bra_e = ket_e.dag()
bra_g = ket_g.dag()
```

Code 3. Definition of the bra states from the ket states using `.dag()`.

Moreover, operators are defined by specifying their matrix representation within `Qobj()`. Therefore, once a basis representation is chosen, it is important to use the same basis consistently in the definition of every operator and state. The lowering operator of the quantum emitter, $\hat{\sigma}_- = |g\rangle\langle e|$, which transforms the excited state into the ground state (i.e., $\hat{\sigma}_-|e\rangle = |g\rangle$), can be defined as detailed in Code 4.

```
sigma_minus = Qobj([[0, 0], [1, 0]])
```

Code 4. Definition of the lowering operator $\hat{\sigma}_- = |g\rangle\langle e|$ using Qobj.

Notably, QuTiP provides built-in functions for many commonly used operators. For example, the lowering operator can be also defined as described in Code 5.

```
sigma_minus = sigmam()
```

Code 5. Definition of the lowering operator $\hat{\sigma}_- = |g\rangle\langle e|$ using the built-in function `sigmam()` from QuTiP.

Other examples of built-in functions to define operators include the raising operator $\hat{\sigma}_+ = |e\rangle\langle g|$, which transforms the ground state into the excited state (i.e., $\hat{\sigma}_+|g\rangle = |e\rangle$), and the Pauli matrices $\hat{\sigma}_x = \hat{\sigma}_+ + \hat{\sigma}_-$, $\hat{\sigma}_y = -i\hat{\sigma}_+ + i\hat{\sigma}_-$, and $\hat{\sigma}_z = \hat{\sigma}_+\hat{\sigma}_- - \hat{\sigma}_-\hat{\sigma}_+$ (Code 6).

```
sigma_plus = sigmap() #Built-in function to define the raising operator
sigma_x = sigmax() # Built-in function to define the sigma_x Pauli matrix
sigma_y = sigmay() #Built-in function to define the sigma_y Pauli matrix
sigma_z = sigmaz() #Built-in function to define the sigma_z Pauli matrix
```

Code 6. Definition of the raising operator and the Pauli matrices using built-in functions from QuTiP.

In addition, QuTiP provides a wide range of built-in states and operators for bosonic systems, which will be outlined in Section 7.

We emphasize that the codes throughout this tutorial are designed to be executed sequentially, with variables, operators, and imported libraries reused throughout the tutorial. For example, the NumPy package is imported only in Code 1, while the QuTiP package is imported only in Code 2. Similarly, several relevant operators and parameters are defined only once and subsequently used in later codes.

3. Spontaneous emission from a quantum emitter

In this section, we describe the interaction between the quantum emitter and the electromagnetic vacuum field. We briefly review the concept of spontaneous emission using both the Wigner-Weisskopf approximation and the Lindblad master equation formalism, the latter constituting a central framework for modelling the dynamics of open quantum systems in QuTiP. We then describe how to implement the dissipative dynamics of the quantum emitter within QuTiP and perform numerical simulations that illustrate the spontaneous emission of light from the quantum emitter. Further details on the analytical derivations presented in this section can be found in Refs. [17–20].

3.a. Interaction between the electromagnetic vacuum field and a quantum emitter

The electromagnetic radiative properties of a quantum emitter in free space are governed by its interaction with the electromagnetic vacuum field. This field corresponds to the quantized electromagnetic field in free space and can be described as a continuum of independent electromagnetic modes that exist even in the absence of photons.

The dynamics of the composite system (i.e., the emitter together with the vacuum field) are determined by the Schrödinger equation and are therefore governed by the total Hamiltonian, which we describe next. On the one hand, the Hamiltonian of the quantum emitter, setting the origin of energy at the ground state, is given by

$$\hat{H}_{QE} = \hbar\omega_{eg}\hat{\sigma}_+\hat{\sigma}_- = \hbar\omega_{eg}|e\rangle\langle e|. \quad (1)$$

On the other hand, the electromagnetic vacuum field is described by a continuum set of quantum harmonic oscillators, each associated with a different electromagnetic mode with wavevector $\mathbf{k}$ and polarization unit vector $\mathbf{e}_{ks}$ (with $s = 1, 2$ labelling two possible orthogonal polarization directions perpendicular to $\mathbf{k}$). Thus, the electromagnetic vacuum field can be described by the Hamiltonian

$$\hat{H}_V = \sum_{\mathbf{k}, s} \hbar\omega_k \hat{a}_{ks}^\dagger \hat{a}_{ks}, \quad (2)$$

where $\hat{a}_{ks}^\dagger$ and $\hat{a}_{ks}$ are the creation and annihilation operators of photons in mode $(\mathbf{k}, s)$, and $\omega_k = ck$ is the mode frequency, with c the speed of light in vacuum and $k = |\mathbf{k}|$. The action of these operators on the Fock state $|n\rangle_{ks}$, representing a state with n photons in mode $(\mathbf{k}, s)$, is given by $\hat{a}_{ks}^\dagger|n\rangle_{ks} = \sqrt{n+1}|n+1\rangle_{ks}$ (which increases the photon occupation number of the mode by one) and $\hat{a}_{ks}|n\rangle_{ks} = \sqrt{n}|n-1\rangle_{ks}$ (which decreases the photon occupation number by one). The photon operators satisfy the commutation relation $[\hat{a}_{ks}, \hat{a}_{k's'}^\dagger] = \delta_{\mathbf{k}, \mathbf{k}'}\delta_{s, s'}$, and $[\hat{a}_{ks}, \hat{a}_{k's'}] = 0$.

The interaction between the vacuum field and the quantum emitter is conveniently described in its multipolar form, obtained after applying the Power-Zienau transformation. This Hamiltonian is given by

$\hat{H}_I = -\hat{\boldsymbol{\mu}} \cdot \hat{\mathbf{E}}(\mathbf{r}_{QE})$, where $\hat{\boldsymbol{\mu}}$ is the dipole moment operator of the emitter and $\hat{\mathbf{E}}(\mathbf{r}_{QE})$ is the transverse electric field operator at the position of the quantum emitter, $\mathbf{r}_{QE}$. Throughout this tutorial, we assume that the emitter possesses only a transition (or induced) dipole moment. In other words, only the off-diagonal matrix elements $\boldsymbol{\mu} = \langle g | \hat{\boldsymbol{\mu}} | e \rangle$ and $\boldsymbol{\mu}^* = \langle e | \hat{\boldsymbol{\mu}} | g \rangle$ are non-zero, while the diagonal matrix elements $\langle g | \hat{\boldsymbol{\mu}} | g \rangle$ and $\langle e | \hat{\boldsymbol{\mu}} | e \rangle$, corresponding to permanent dipole moments, are assumed to vanish. This approximation accurately describes many common two-level quantum emitters. However, for systems such as polar molecules [21,22] or asymmetric quantum dots [23], the diagonal matrix elements of $\hat{\boldsymbol{\mu}}$ cannot be neglected. In such cases, the theoretical framework must be extended to include permanent dipole moments [24–26], a topic that lies beyond the scope of this tutorial.

Moreover, neglecting the terms that do not conserve the total number of excitations in $\hat{H}_I$ —a procedure known as the Rotating-Wave Approximation (RWA), since these terms correspond to rapidly oscillating contributions— the interaction Hamiltonian can be written as

$$\hat{H}_I = \hbar \sum_{\mathbf{k},s} (g_{\mathbf{k}s} \hat{a}_{\mathbf{k}s} \hat{\sigma}_+ + g_{\mathbf{k}s}^* \hat{a}_{\mathbf{k}s}^\dagger \hat{\sigma}_-), \quad (3)$$

where the coupling coefficient is given by $g_{\mathbf{k}s} = -i \boldsymbol{\mu}^* \cdot \mathbf{e}_{\mathbf{k}s} \exp(i\mathbf{k} \cdot \mathbf{r}) \sqrt{\omega_{\mathbf{k}} / (2\varepsilon_0 \hbar \mathcal{V})}$, with $\boldsymbol{\mu} = \langle g | \hat{\boldsymbol{\mu}} | e \rangle$ the transition dipole moment, ε_0 the vacuum permittivity, and $\mathcal{V}$ the quantization volume. The first term in Eq. (3) describes the absorption of a photon accompanied by the excitation of the emitter, while the second term corresponds to photon emission together with the de-excitation of the emitter.

3.b. The Wigner-Weisskopf approximation

We next analyze the spontaneous emission of photons by the quantum emitter, which arises from its interaction with the electromagnetic vacuum field. To this end, we derive the time evolution of an initial state in which the emitter is in its excited state and no photons are present in the field. This initial state can be written as $|\psi(t=0)\rangle = |e, 0\rangle = |e\rangle \otimes |0\rangle$, where $\otimes$ is the tensor product and $|0\rangle$ the vacuum state. To derive the time evolution of this state, we use the Schrödinger equation in the interaction picture [27], which is given by

$$\frac{d}{dt} |\psi(t)\rangle = -\frac{i}{\hbar} \hat{H}_I(t) |\psi(t)\rangle, \quad (4)$$

where $|\psi(t)\rangle$ is the state at time t in the interaction picture and

$$\begin{aligned} \hat{H}_I(t) &= \exp[it(\hat{H}_{QE} + \hat{H}_V)/\hbar] \hat{H}_I \exp[-it(\hat{H}_{QE} + \hat{H}_V)/\hbar] \\ &= \hbar \sum_{\mathbf{k},s} (g_{\mathbf{k}s} \hat{a}_{\mathbf{k}s} \hat{\sigma}_+ \exp[it(\omega_{eg} - \omega_{\mathbf{k}})] + g_{\mathbf{k}s}^* \hat{a}_{\mathbf{k}s}^\dagger \hat{\sigma}_- \exp[-it(\omega_{eg} - \omega_{\mathbf{k}})]) \end{aligned} \quad (5)$$

is the interaction Hamiltonian in the interaction picture. The interaction picture simplifies the analytical calculations in this section, as only the interaction Hamiltonian plays an explicit role in Eq. (4), whereas $\hat{H}_{QE}$ and $\hat{H}_V$ only provide the exponential functions in the last line of Eq. (5).

As the RWA is considered, the evolution from the initial state necessarily conserves the number of excitations in total (between photons and emitter excitations). Consequently, the system can only evolve into states containing either an excited emitter with no photons or the emitter in its ground state with a single photon in one of the electromagnetic-field modes. Thus, the state can be written as

$$|\psi(t)\rangle = c^{(e)}(t) |e, 0\rangle + \sum_{\mathbf{k},s} c_{\mathbf{k}s}^{(g)}(t) \hat{a}_{\mathbf{k}s}^\dagger |g, 0\rangle, \quad (6)$$

where $\hat{a}_{\mathbf{k}s}^\dagger |g, 0\rangle$ corresponds to a state in which the emitter is in the ground state and a photon has been created in the electromagnetic mode $(\mathbf{k}, s)$. The probability amplitudes $c^{(e)}(t)$ and $c_{\mathbf{k}s}^{(g)}(t)$ are unknown at this stage. To obtain these probability amplitudes, we next use the Wigner-Weisskopf approximation [28,29].

We substitute Eq. (6) into both sides of Eq. (4), which yields the following equations of motion for the probability amplitudes:

$$\frac{d}{dt}c^{(e)}(t) = -i \sum_{k,s} c_{ks}^{(g)}(t) g_{ks} \exp[i(\omega_{eg} - \omega_k)t], \quad (7A)$$

$$\frac{d}{dt}c_{ks}^{(g)}(t) = -ic^{(e)}(t)g_{ks}^* \exp[-it(\omega_{eg} - \omega_k)]. \quad (7B)$$

Now, Eq. (7B) is formally integrated in time, and the resulting integral expression for $c_{ks}^{(g)}(t)$ is substituted into Eq. (7A), leading to the following equation:

$$\frac{d}{dt}c^{(e)}(t) = - \int_0^t dt' \sum_{k,s} c^{(e)}(t') |g_{ks}|^2 \exp[i(\omega_{eg} - \omega_k)(t - t')]. \quad (8)$$

The right-hand side of this equation can be evaluated by replacing the sum over $\mathbf{k}$ with an integral, which is justified because free space supports a very large number of closely spaced electromagnetic modes, and by assuming Markovian dynamics for the emission process [20,30,31], meaning that the electromagnetic vacuum does not retain memory of previous emitter dynamics. As a consequence, one obtains that $c^{(e)}(t)$ decays exponentially [20]:

$$c^{(e)}(t) = c^{(e)}(t=0) \exp[-\gamma_{eg}t/2], \quad (9)$$

where the decay rate is given by

$$\gamma_{eg} = \frac{\omega_{eg}^3 |\boldsymbol{\mu}|^2}{3\pi\epsilon_0 \hbar c^3}. \quad (10)$$

This coefficient γ_{eg} is known as spontaneous emission rate and corresponds to the inverse of the lifetime of the excited state of the emitter, as the probability of finding the state still excited at time t is $|c^{(e)}(t)|^2 = \exp[-\gamma_{eg}t]$, where we have considered the initial condition $c^{(e)}(t=0) = 1$. The vacuum Lamb shift has been neglected, as its correction to the electronic transition frequency is negligible in typical quantum emitters.

We therefore conclude that the population of the excited state of a quantum emitter decays exponentially, with a decay rate γ_{eg} determined by the transition frequency ω_{eg} and the magnitude of the transition dipole moment $\boldsymbol{\mu}$ [Eq. (10)]. The relaxation of the emitter originates from its coupling with the electromagnetic vacuum field, and the exponential decrease of its excited-state population is accompanied by an increase in the population of the ground state, guaranteeing that $\langle\psi(t)|\psi(t)\rangle = 1$, meaning that a photon is created when the emitter relaxes. Specifically, substituting Eq. (9) into Eq. (7B), we obtain

$$|c_{ks}^{(g)}(t)|^2 = \frac{|g_{ks}|^2}{(\gamma_{eg}/2)^2 + (\omega_{eg} - \omega_k)^2} \left[1 - \exp\left\{-t\left(\frac{\gamma_{eg}}{2} + i(\omega_{eg} - \omega_k)\right)\right\}\right]^2, \quad (11)$$

where $|g_{ks}|^2 \propto \omega_k \approx \omega_{eg}$, with the approximation holding for $\omega_{eg} \gg \gamma_{eg}$. Thus, the relaxation of the excited state leads to the spontaneous emission of a photon, with a probability density for the electromagnetic mode in which the photon is created that follows a Lorentzian distribution centered at the transition frequency ω_{eg} (which is thus the most probable emission frequency). The linewidth of this distribution is directly determined by the excited-state lifetime, illustrating the fundamental connection between temporal decay and spectral broadening. Small deviations from purely exponential decay can arise when going beyond the Markovian approximation [32,33]; however, these corrections are negligible (and typically unobservable) in experiments with quantum optical emitters, thereby validating the use of this approximation.

3.c. The Lindblad master equation

Comparing the analytical expressions derived within the Wigner–Weisskopf approximation with numerical brute-force calculations comprising the full Hilbert space is challenging, as the electromagnetic vacuum field consists of an infinite set of harmonic oscillators. A key tool in this context is the Markovian master equation [27,34], which enables a reduction to the Hilbert space of a subsystem—treated as the system of interest (in this case, the quantum emitter)—by tracing out the degrees of freedom of another subsystem, regarded as the environment (in this case, the vacuum field).

The validity of the Markovian master equation relies on two main assumptions. First, the Born approximation requires weak coupling between the system of interest (S) and the environment (E). Second, the Markov approximation assumes a rapid decay of environmental two-time correlations compared to the characteristic timescale of the system dynamics. Under these conditions, the evolution of the state of the system, described by the density matrix $\hat{\rho}_S$, is governed in the interaction picture by the following equation:

$$\frac{d}{dt}\hat{\rho}_S(t) = -\frac{1}{\hbar^2} \int_0^\infty dt' Tr_E \left[\hat{H}_I(t), [\hat{H}_I(t-t'), \hat{\rho}_S(t) \otimes \hat{\rho}_E] \right], \quad (12)$$

where Tr_E is the partial trace with respect to the environment Hilbert space, and $\hat{\rho}_E$ is the density matrix of the environment, which is assumed constant in time because the weak interaction with the system of interest does not affect its initial state. For example, $\hat{\rho}_E$ can represent a thermal state.

The application of this framework to the interaction between the quantum emitter, considered as the system of interest, and the electromagnetic vacuum field, considered as the environment, leads to the following equation governing the reduced dynamics of the emitter in the Schrödinger picture:

$$\frac{d}{dt}\hat{\rho}_{QE}(t) = -\frac{i}{\hbar} [\hat{H}_{QE}, \hat{\rho}_{QE}(t)] + \frac{\gamma_{eg}}{2} \mathcal{D}[\hat{\sigma}_-] \hat{\rho}_{QE}(t), \quad (13)$$

where $\mathcal{D}$ is a superoperator (i.e., it transforms operators into operators) known as Lindblad dissipator, whose action on an arbitrary Lindblad operator $\hat{O}$ and an arbitrary density matrix $\hat{\rho}$ is defined as

$$\mathcal{D}[\hat{O}]\hat{\rho} = 2\hat{O}\hat{\rho}\hat{O}^\dagger - \hat{O}^\dagger\hat{O}\hat{\rho} - \hat{\rho}\hat{O}^\dagger\hat{O}. \quad (14)$$

The first term on the right-hand side of Eq. (13) describes the coherent dynamics of the emitter, determined by the commutator of the Hamiltonian and the density matrix (i.e., it yields the Von Neumann equation that governs the dynamics of closed quantum systems [27]). The vacuum Lamb shift correction has again been neglected. The second term on the right-hand side of Eq. (13) accounts for the incoherent dynamics of the emitter due to its interaction with the environment. Specifically, this term describes the spontaneous emission of photons from the emitter, and thus the loss of population from the excited state of the emitter. Equation (14) is directly computed in the two-dimensional Hilbert space of the emitter, and it can thus be solved numerically in short computation time. We solve it numerically in the next section using QuTiP to illustrate the exponential decay obtained analytically in Section 3.2.b using the Wigner-Weisskopf approximation.

Importantly, the master-equation formalism provides a powerful framework for describing the dynamics of the emitter in more complex scenarios, such as when the emitter is driven by a laser, as discussed in Section 4, or when it interacts with an optical cavity, as discussed in Section 7.

3.d. Numerical simulations using QuTiP

To numerically solve the evolution of the density matrix $\hat{\rho}_{QE}(t)$ dictated by the master equation (13), we employ the function `mesolve(H, rho0, tlist, c_ops)` from QuTiP. The key inputs of this function are the Hamiltonian of the system (H), the initial state (ρ_0), a time array specifying the evaluation points ($tlist$), and a list of operators (c_ops) that accounts for the Lindblad dissipators. More specifically, the list c_ops must include one element for each Lindblad dissipator $\mathcal{D}[\hat{O}]$ contributing to the incoherent dynamics. Each of these elements is constructed as the product of the corresponding Lindblad operator $\hat{O}$ and the square root of its associated decay rate. In the case of the master equation (13), the system is subject to a single dissipative process (spontaneous emission), and therefore c_ops contains only one element, given by $\sqrt{\gamma_{eg}}\hat{\sigma}_-$. Further, we assume that the emitter is initially in the excited state (i.e., the initial state is $|e\rangle\langle e| = \hat{\sigma}_+\hat{\sigma}_-$), in order to observe the exponential decay of the excited-state population at a rate γ_{eg} due to spontaneous photon emission.

As a reference, we consider a quantum emitter in vacuum with a transition frequency in the visible range, corresponding to a wavelength $\lambda = 600$ nm, and thus $\omega_{eg} = 2\pi c/\lambda$. Additionally, we assume an excited-state lifetime of $1/\gamma_{eg} = 10$ ns, which is characteristic of typical organic molecules [1,8]. With these parameters, the inputs to `mesolve` can be constructed as described in Code 7. We note that these input parameters, together with the required physical constants, are defined in SI units in this code. Additional parameters introduced in subsequent codes are generally expressed in units of γ_{eg} .

```
c = 3e8 # Speed of light (SI units)
wavelength = 600e-9 # Wavelength of the two-level transition
omega_eg = 2 * numpy.pi * c / wavelength # Transition frequency
gamma_eg = 1 / (10e-9) # Spontaneous emission rate
H = omega_eg * sigma_minus.dag() * sigma_minus # Hamiltonian (in units of  $\hbar$ ) in Eq. (1)
c_ops = [numpy.sqrt(gamma_eg) * sigma_minus] # List of Lindblad operators
tlist = numpy.linspace(0, 100, 300)*1e-9 # Time grid for evaluation
rho0 = sigma_minus.dag() * sigma_minus # Initial state (excited state)
result = mesolve(H, rho0, tlist, c_ops)
```

Code 7. Calculation of the time evolution of the quantum-emitter density matrix under the master equation (13) using `mesolve`.

Next, we can access the list of `Qobj` instances corresponding to the density matrix at each time in `tlist` via `result.states`. This allows us to directly reconstruct the time evolution of the excited- and ground-state populations and plot them (Code 8).

```
population_e = [] #This list will store the time evolution of the excited-state population
population_g = [] #This list will store the time evolution of the ground-state population
for rho in result.states:
    population_e.append(rho[0,0].real)
    population_g.append(rho[1,1].real)
#We next plot the results
import matplotlib.pyplot as plt #Import plotting library
plt.figure(figsize=(6, 4))
plt.plot(tlist * 1e9, population_e, lw=2, label=r"$P_e$")
plt.plot(tlist * 1e9, population_g, lw=2, ls="--", label=r"$P_g$")
plt.xlabel(r"$t \backslash$, [ns]")
plt.ylabel("Population")
plt.legend(frameon=False)
plt.show() #The resulting plot is shown in Fig. 2a
```

Code 8. Extraction and plotting of the excited- and ground-state populations from the calculation performed in Code 7. The results are shown in Fig. 2a.

Figure 2a shows the resulting excited-state population (solid blue line) and ground-state population (dashed orange line). The excited-state population decreases exponentially from 1 to 0, which is compensated by an exponential increase in the ground-state population from 0 to 1. This behavior is due to the spontaneous emission of a photon from the excited state resulting from the interaction with the electromagnetic vacuum field acting as a reservoir, consistently with the analytical results obtained with the Wigner-Weisskopf approximation in Section 3.b.

Codes 7 and 8 can be easily adapted to gain further insight into spontaneous photon emission and into the relaxation dynamics of the excited state of the emitter. For instance, the time evolution of the excited-state population can be systematically analyzed for different values of the decay rate γ_{eg} , as shown in Fig. 2b for $1/\gamma_{eg} = 1$ ns (solid blue line), $1/\gamma_{eg} = 10$ ns (solid orange line), $1/\gamma_{eg} = 20$ ns (solid green line), and $1/\gamma_{eg} = 50$ ns (solid red line). Figure 2b illustrates faster relaxation associated with shorter lifetimes (i.e., larger spontaneous emission rates). Additional simulations could include a systematic variation of the

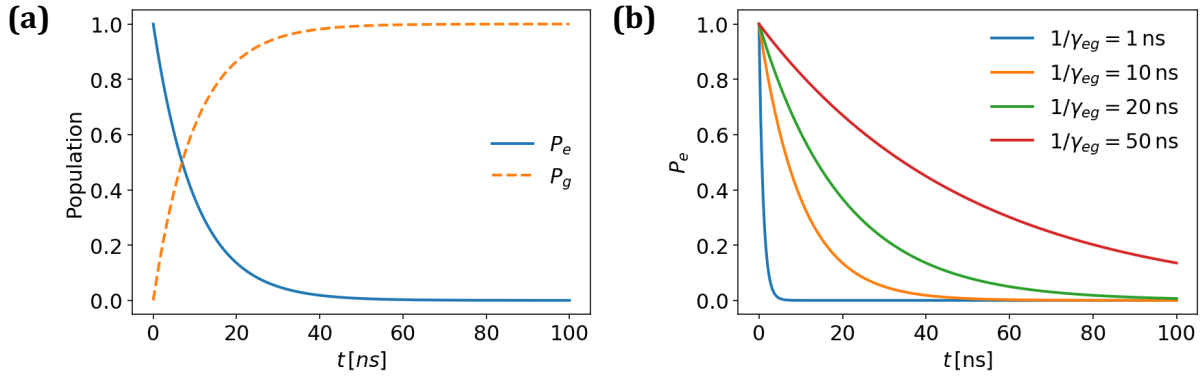


Fig. 2. Relaxation of an undriven quantum emitter. (a) Time evolution of the excited-state population (solid blue line) and the ground-state population (dashed orange line) for $1/\gamma_{eg} = 10$ ns. This plot is obtained using Codes 7 and 8. (b) Comparison of the excited-state population dynamics for different spontaneous emission lifetimes: $1/\gamma_{eg} = 1$ ns (solid blue line), $1/\gamma_{eg} = 10$ ns (solid orange line), $1/\gamma_{eg} = 20$ ns (solid green line), and $1/\gamma_{eg} = 50$ ns (solid red line). The transition frequency corresponds to a vacuum wavelength of 600 nm.

transition frequency and of the initial excited-state population, as well as the consideration of initial states with non-zero coherence between the ground and excited states.

Furthermore, we emphasize that the built-in function `mesolve` in QuTiP can be used to numerically simulate the dynamics of a broad class of open quantum systems, provided that they can be described by a Lindblad master equation. As such, it provides a flexible and powerful framework for investigating dissipative quantum dynamics in a wide variety of nanophotonic and quantum optical systems.

4. Driving a quantum emitter with a laser

In this section, we describe the interaction between a quantum emitter and a coherent light source, such as a laser. Specifically, we consider a continuous-wave laser with frequency ω_L close to resonance with the emitter transition frequency, and with polarization unit vector $\mathbf{e}_L$, such that its electric field can be described as a plane wave $\mathbf{E}_L(\mathbf{r}, t) = E_L(\mathbf{r}) \cos(\omega_L t) \mathbf{e}_L$. The case of incoherent driving is discussed in Appendix B.

As in the case of the vacuum-emitter interaction discussed in Section 3.a, the Schrödinger-picture Hamiltonian describing the interaction between the laser and the emitter can again be written in the multipolar form. In this case, the interaction Hamiltonian is given by the product of the emitter dipole moment operator and the laser electric field, namely $\hat{H}_L(t) = -\hat{\boldsymbol{\mu}} \cdot \mathbf{E}_L(\mathbf{r}_{QE}, t)$ [17,18,20,27]. In the absence of driving, the lowering and raising operators evolve in the Heisenberg picture as $\exp(-i\omega_{eg}t)$ and $\exp(i\omega_{eg}t)$, respectively. Consequently, the terms $\hat{\sigma}_- \exp(-i\omega_L t)$ and $\hat{\sigma}_+ \exp(i\omega_L t)$ in $\hat{H}_L(t)$ oscillate at frequencies close to $\omega_L + \omega_{eg}$. Under near-resonant driving, these terms oscillate much more rapidly than the counter-rotating terms $\hat{\sigma}_- \exp(i\omega_L t)$ and $\hat{\sigma}_+ \exp(-i\omega_L t)$, which oscillate at frequencies close to $|\omega_L - \omega_{eg}|$. The rapidly oscillating terms average out over the timescale of the system dynamics and can be neglected within the rotating-wave approximation (RWA). The interaction Hamiltonian can be approximated as

$$\hat{H}_L(t) = -\frac{\hbar}{2} [\Omega \hat{\sigma}_- \exp(i\omega_L t) + \Omega^* \hat{\sigma}_+ \exp(-i\omega_L t)], \quad (15)$$

where we have introduced the so-called Rabi frequency $\Omega = E_L(\mathbf{r}_{QE}) \boldsymbol{\mu} \cdot \mathbf{e}_L / \hbar$, which is proportional to the amplitude of the laser field (i.e., to the square root of the laser intensity). We note that the RWA breaks down under very strong driving, when the Rabi frequency becomes comparable to the transition frequency, making the counter-rotating terms non-negligible and giving rise to observable effects such as the Bloch-Siegert shift [35–38]. A discussion of this regime is beyond the scope of this tutorial.

The total Hamiltonian describing the coherent dynamics of the driven emitter is therefore given by the sum of the bare Hamiltonian $\hat{H}_{QE}$ [Eq. (1)] and the driving Hamiltonian $\hat{H}_L(t)$ [Eq. (15)]. As a result, the laser modifies the eigenstates of the system, which become

$$|\Omega_+\rangle = \sin \Theta |g\rangle + \cos \Theta |e\rangle, \quad |\Omega_-\rangle = \cos \Theta |g\rangle - \sin \Theta |e\rangle, \quad (16)$$

where the angle $\Theta \in [0, \pi/2]$ is defined by $\tan(2\Theta) = |\Omega|/\omega_{eg}$. These states, known as dressed states, have an energy splitting $\hbar\sqrt{\omega_{eg}^2 + |\Omega|^2}$, which increases with the laser intensity. In the absence of driving ($\Omega \rightarrow 0$), one recovers the bare states $|\Omega_+\rangle \rightarrow |e\rangle$ and $|\Omega_-\rangle \rightarrow |g\rangle$, with energy splitting $\hbar\omega_{eg}$.

We next discuss two relevant phenomena associated with laser driving. First, we analyze the emergence of oscillations in the time evolution of the excited- and ground-state populations, known as Rabi oscillations. Second, we examine the saturation of the steady-state excited-state population as the laser intensity increases.

4.a. Rabi oscillations

We now investigate the time evolution of the excited-state population, as in Fig. 2, but including the effect of laser driving. To facilitate the numerical calculations, we first perform a unitary transformation that allows us to work with a time-independent Hamiltonian. This transformation can be interpreted as moving to a reference frame rotating at the laser frequency. It is given by $\hat{U} = \exp(i\omega_L t \hat{\sigma}_+ \hat{\sigma}_-)$, and transforms the total Hamiltonian $\hat{H} = \hat{H}_{QE} + \hat{H}_L(t)$ as [20]

$$\hat{H}' = i\hbar \frac{d\hat{U}}{dt} \hat{U}^\dagger + \hat{U} \hat{H} \hat{U}^\dagger = \hbar\Delta_{eg} \hat{\sigma}_+ \hat{\sigma}_- - \frac{\hbar}{2} (\Omega \hat{\sigma}_- + \Omega^* \hat{\sigma}_+), \quad (17)$$

where we have introduced the detuning $\Delta_{eg} = \omega_{eg} - \omega_L$. In this rotating frame, the dressed states can again be written as in Eq. (16), but the angle Θ is now determined by $\tan(2\Theta) = |\Omega|/\Delta_{eg}$. The energy splitting between the dressed states becomes $\hbar\sqrt{\Delta_{eg}^2 + |\Omega|^2}$, which reduces to $\hbar|\Omega|$ under resonant driving ($\Delta_{eg} = 0$).

The complete dissipative dynamics is then described by the master equation:

$$\frac{d}{dt} \hat{\rho}_{QE}(t) = -\frac{i}{\hbar} \left[\hbar\Delta_{eg} \hat{\sigma}_+ \hat{\sigma}_- - \frac{\hbar}{2} (\Omega \hat{\sigma}_- + \Omega^* \hat{\sigma}_+), \hat{\rho}_{QE}(t) \right] + \frac{\gamma_{eg}}{2} \mathcal{D}[\hat{\sigma}_-] \hat{\rho}_{QE}(t). \quad (18)$$

The codes developed in Section 3.d can be readily adapted to include the laser driving in this rotating frame (Code 9).

Figure 3a shows the time evolution of the bare-state populations $|e\rangle$ (solid blue line) and $|g\rangle$ (dashed orange line), for an emitter initially prepared in the excited state $|e\rangle$ and driven by a resonant laser ($\omega_L = \omega_{eg}$), with Rabi frequency $\Omega = 5\gamma_{eg}$. The populations exhibit oscillations, known as Rabi oscillations [17,20,39], which arise because the bare states $|e\rangle$ and $|g\rangle$ are not eigenstates of the driven system. Instead, $|e\rangle$ and $|g\rangle$ are superpositions of dressed states [Eq. (16)], so their populations oscillate in time as these contributions combine differently. The oscillations in Fig. 3a occur at a frequency Ω , which corresponds to the energy splitting between the dressed states in the rotating frame under resonant driving. Superimposed on this oscillatory behavior, the excited-state population exhibits an overall decay. However, in contrast to the undriven case (Fig. 2), the system reaches a steady state with a non-zero excited-state population.

Code 9 can again be adapted to gain further insight into the dynamics of the driven emitter and the Rabi oscillations. For example, Fig. 3b shows the excited-state population for different values of Rabi frequency: $\Omega = 5\gamma_{eg}$ (solid blue line), $\Omega = 2\gamma_{eg}$ (solid orange line), and $\Omega = \gamma_{eg}/10$ (solid green line). At $\Omega = 2\gamma_{eg}$, the Rabi oscillations have a clearly larger period (i.e., a lower Rabi frequency) compared to the $\Omega = 5\gamma_{eg}$ case, and the steady-state population is slightly smaller. Notably, at $\Omega = \gamma_{eg}/10$ we recover an undriven-like behavior. Importantly, the code can be easily adapted to explore the case in which the emitter is driven off resonance (leading to oscillations at the so-called generalized Rabi frequency, see Appendix A), or different initial states, for instance.

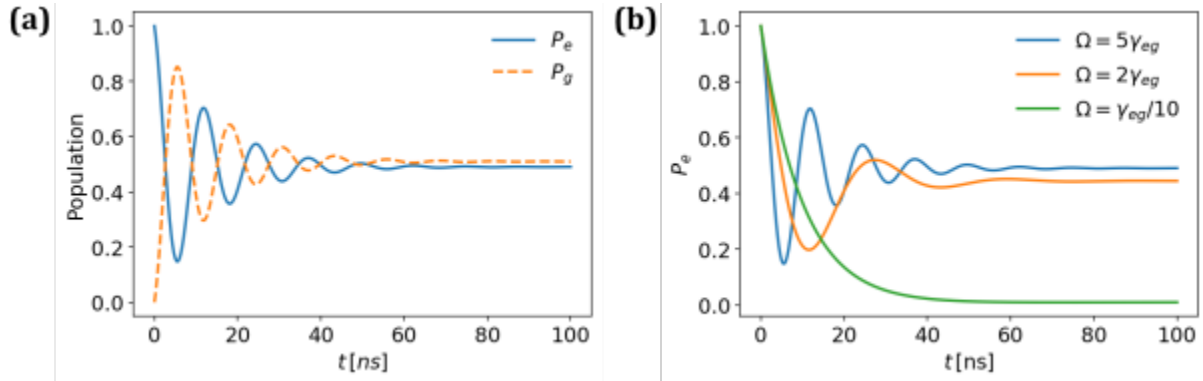


Fig. 3. Time evolution of a driven quantum emitter. (a) Time evolution of the excited-state population (solid blue line) and the ground-state population (dashed orange line) for $\Omega = 5\gamma_{eg}$. This plot is obtained using Code 9. (b) Comparison of the excited-state population dynamics for different values of Rabi frequency: $\Omega = 5\gamma_{eg}$ (solid blue line), $\Omega = 2\gamma_{eg}$ (solid orange line), and $\Omega = \gamma_{eg}/10$ (solid green line). The transition frequency corresponds to a vacuum wavelength of 600 nm and lifetime $1/\gamma_{eg} = 10$ ns. The laser is resonantly tuned to the emitter transition frequency.

```

omega_L = omega_eg #We focus on resonant driving
detuning = omega_eg-omega_L
Rabi_freq = 5*gamma_eg
H = detuning * sigma_minus.dag() * sigma_minus - 0.5*Rabi_freq*(sigma_minus+sigma_minus.dag())
c_ops = [numpy.sqrt(gamma_eg) * sigma_minus] # List of Lindblad operators
tlist = numpy.linspace(0, 100, 300)*1e-9 # Time grid for evaluation
rho0 = sigma_minus.dag() * sigma_minus # Initial state (excited state)
result = mesolve(H, rho0, tlist, c_ops)
population_e = [] #This list will store the time evolution of the excited-state population
population_g = [] #This list will store the time evolution of the ground-state population
for rho in result.states:
    population_e.append(rho[0,0].real)
    population_g.append(rho[1,1].real)
plt.figure(figsize=(6, 4))
plt.plot(tlist*1e9, population_e, lw=2, label=r"$P_e$")
plt.plot(tlist*1e9, population_g, lw=2, ls="--", label=r"$P_g$")
plt.xlabel(r"$t$, [ns] $")
plt.ylabel("Population")
plt.legend(frameon=False)
plt.show() #The resulting plot is shown in Fig 3a

```

Code 9. Calculation of time evolution of the excited- and ground-state populations of the driven emitter in the rotating frame at the laser frequency. Specifically, mesolve is used to solve master equation (18). The result is plotted in Fig. 3a.

4.b. Emitter saturation

We next analyze the behavior of the steady-state (ss) excited-state population for increasing values of the Rabi frequency (i.e., driving strength). To this end, we use the function `steadystate(H, c_ops)` from QuTiP, which returns a Qobj corresponding to the steady-state density matrix. This function requires only the Hamiltonian (H) and the list `c_ops` accounting for the Lindblad dissipators. We can thus easily adapt the previous codes to analyze the dependence of the steady-state excited-state population on Ω (Code 10).

Figure 4a shows the behavior of the steady-state excited-state population (solid blue line) as a function of the Rabi frequency, together with the steady-state ground-state population (dashed orange line), again for the case in which the laser is tuned resonantly to the emitter transition ($\Delta_{eg} = 0$). We observe that the excited-state population ranges from approximately 0, in the limit of no driving, to 0.5 under strong driving. Thus, the steady-state excited-state population of the emitter saturates as the driving strength increases

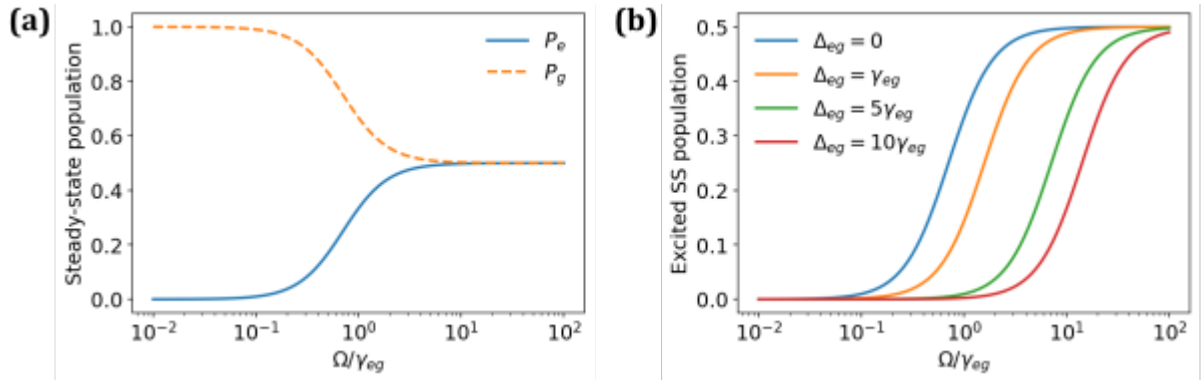


Fig. 4. Behavior of the steady-state population of the excited state of a driven quantum emitter as a function of the driving strength. (a) Steady-state excited (solid blue line) and ground (dashed orange line) populations as a function of the Rabi frequency Ω (normalized by γ_{eg}). The laser is resonantly tuned to the two-level transition frequency. Code 10 is used. (b) Comparison of the behavior of the steady-state population of excited-state for different values of the laser detuning ($\Delta_{eg} = \omega_{eg} - \omega_L$): $\Delta_{eg} = 0$ (solid blue line), $\Delta_{eg} = \gamma_{eg}$ (solid orange line), $\Delta_{eg} = 5\gamma_{eg}$ (solid green line), and $\Delta_{eg} = 10\gamma_{eg}$ (solid red line). The transition frequency used corresponds to a vacuum wavelength of 600 nm and the lifetime to $1/\gamma_{eg} = 10$ ns.

```
Rabi_list = numpy.logspace(-2, 2, 100)* gamma_eg #Array of Rabi frequencies
population_e_ss = [] # List to store steady-state excited-state population
population_g_ss = [] # List to store steady-state excited-state population
for Rabi_freq in Rabi_list: # Loop over Rabi frequencies
    H = detuning*sigma_minus.dag()*sigma_minus - 0.5 * Rabi_freq * (sigma_minus + sigma_minus.dag())
    rho_ss = steadystate(H, c_ops) # Steady-state density matrix
    population_e_ss.append(rho_ss[0,0].real) # Excited-state population
    population_g_ss.append(rho_ss[1,1].real) # Ground-state population
plt.figure(figsize=(6,4))
plt.plot(Rabi_list / gamma_eg, population_e_ss, lw=2, label="$P_e$")
plt.plot(Rabi_list / gamma_eg, population_g_ss, lw=2, ls="--", label="$P_g$")
plt.xscale("log")
plt.xlabel(r"$\Omega/\gamma_{eg}$")
plt.ylabel(r"Steady-state population")
plt.legend(frameon=False)
plt.show() #The resulting plot is shown in Fig. 4a
```

Code 10. Calculation of the steady-state populations using steadystate and their dependence on Ω . The result is plotted in Fig. 4a.

and cannot exceed the steady-state ground-state population. This behavior is known as the saturation of the quantum emitter [17].

Code 10 can be adapted to analyze the impact of other parameters in the steady-state populations and thus gain further insight into the concept of emitter saturation. For example, Fig. 4b shows the steady-state excited-state population for different values of the laser detuning, namely $\Delta_{eg} = 0$ (solid blue line), $\Delta_{eg} = \gamma_{eg}$ (solid orange line), $\Delta_{eg} = 5\gamma_{eg}$ (solid green line), and $\Delta_{eg} = 10\gamma_{eg}$ (solid red line). We observe that, in all cases, the excited-state population saturates at the same value (0.5). However, the value of the Rabi frequency required to reach the saturation regime increases with the laser detuning, since the laser becomes less efficient at driving the transition when its frequency moves away from resonance with the emitter.

5. Emission spectrum and the Mollow triplet

We have so far focused on characterizing the state of the quantum emitter by performing simulations that solve for its density matrix. We next turn to the characterization of the light emitted by this system. A fundamental tool for analyzing the spectral properties of emitted light is the emission spectrum, which reveals the frequencies at which light is preferentially emitted.

The emission spectrum is constructed from the electric field operator, which is conveniently decomposed

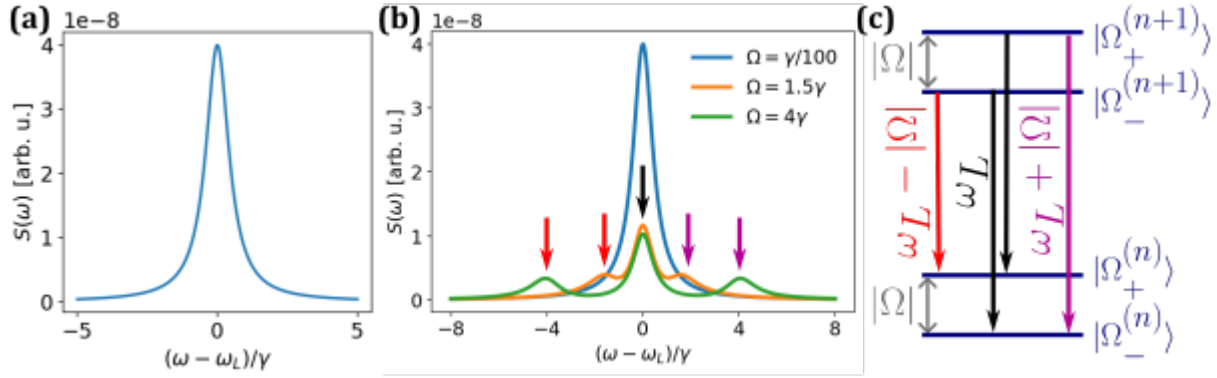


Fig. 5. Emission spectrum of a quantum emitter. (a) Emission spectrum under weak driving ($\Omega = \gamma_{eg}/100$), showing a single peak at $\omega = \omega_L = \omega_{eg}$. The simulation is performed using Code 11. (b) Emission spectra for increasing values of the Rabi frequency: $\Omega = \gamma_{eg}/100$ (solid blue line), $\Omega = 1.5\gamma_{eg}$ (solid orange line), and $\Omega = 4\gamma_{eg}$ (solid green line). For larger values of the Rabi frequency, the spectrum develops the characteristic Mollow triplet, with peaks at $\omega = \omega_L = \omega_{eg}$ (black arrow), $\omega = \omega_L - |\Omega|$ (red arrow), and $\omega = \omega_L + |\Omega|$ (purple arrow). (c) Schematic representation of the dressed-state energy levels, including the number of photons in the laser field, for the n and $n+1$ rungs. In the n -th rung, the dressed states $|\Omega_{\pm}^{(n)}\rangle$ and $|\Omega_{\pm}^{(n+1)}\rangle$ are split by an energy $\hbar|\Omega|$, and the same splitting occurs for the dressed states $|\Omega_{\pm}^{(n+1)}\rangle$ and $|\Omega_{\pm}^{(n+2)}\rangle$ in the $n+1$ rung. Each arrow indicates a different transition associated with one of the peaks in the Mollow triplet shown in panel (b). The transition frequency used in the simulations corresponds to a vacuum wavelength of 600 nm and the lifetime to $1/\gamma_{eg} = 10$ ns. The laser is resonantly tuned to the emitter transition frequency.

as $\hat{E} = \hat{E}^{(+)} + \hat{E}^{(-)}$. Here, $\hat{E}^{(+)}$ is the positive-frequency electric field operator and is proportional to annihilation or lowering operators, while $\hat{E}^{(-)}$ is the negative-frequency electric field operator and is proportional to creation or raising operators. These two operators are Hermitian conjugates of each other, i.e., $(\hat{E}^{(-)})^\dagger = \hat{E}^{(+)}$. The emission spectrum is obtained from the Fourier transform of the temporal correlation function of these operators in the Heisenberg picture. We focus here on the simpler and commonly studied case of the steady-state emission spectrum, which is given by [17,20,27]

$$S(\omega) = \int_{-\infty}^{\infty} d\tau e^{-i\omega\tau} \langle \hat{E}^{(+)}(\tau) \cdot \hat{E}^{(-)}(0) \rangle_{ss}, \quad (19)$$

where the expectation value $\langle \cdot \rangle_{ss}$ is evaluated in the steady state (ss).

The analytical calculation of two-time correlation functions is often facilitated by the use of the quantum regression theorem [27]. This theorem establishes a correspondence between the differential equations governing two-time correlation functions and those governing the time evolution of single-time expectation values. Specifically, let us consider that the equation of motion for the expectation value of an operator $\hat{A}(\tau)$, derived from the corresponding master equation, is given by

$$\frac{d}{d\tau} \langle \hat{A}(\tau) \rangle = \sum_n c_n \langle \hat{C}_n(\tau) \rangle, \quad (20)$$

where $\{\hat{C}_n\}$ is a set of operators, one of which may be $\hat{A}$, and c_n are complex coefficients. The corresponding equation of motion for the two-time correlation function is then

$$\frac{d}{d\tau} \langle \hat{A}(\tau) \hat{B}(0) \rangle_{ss} = \sum_n c_n \langle \hat{C}_n(\tau) \hat{B}(0) \rangle_{ss}, \quad (21)$$

with $\hat{B}$ an arbitrary operator. Thus, the equation of motion for the two-time correlation function can be obtained directly from that of the corresponding single-time expectation values.

QuTiP provides numerical routines for calculating the spectrum $S(\omega)$, some of them based on the quantum regression theorem. Specifically, we can use the `spectrum(H, wlist, c_ops, E_plus, E_minus)` function, whose inputs are the Hamiltonian (H), the frequency grid (`wlist`), the list of Lindblad operators (`c_ops`), the positive-frequency electric field operator in the Schrödinger picture (E_plus), and the negative-frequency electric field operator in the same picture (E_minus).

We next focus on the emission spectrum of the quantum emitter. In the far-field region, the positive-frequency electric field emitted by a quantum emitter can be written as the product of the classical electric field $\mathbf{E}_\mu(\mathbf{r})$ generated by a point-dipole $\boldsymbol{\mu}$ (given by the transition dipole moment of the emitter) and the

lowering operator of the emitter. More specifically, in the Heisenberg picture this operator becomes [17,20]

$$\hat{E}_{QE}^{(+)}(\mathbf{r}, t) = \mathbf{E}_{\mu}(\mathbf{r})\hat{\sigma}_{-}(t), \quad (22)$$

where the classical field satisfies $\mathbf{E}_{\mu}(\mathbf{r}) \propto \mathbf{e}_R \times (\mathbf{e}_R \times \boldsymbol{\mu})$, with $\mathbf{e}_R = (\mathbf{r} - \mathbf{r}_{QE})/R$ and $R = |\mathbf{r} - \mathbf{r}_{QE}|$ the distance between the emitter and the detection position. For simplicity, we have omitted the retardation time contribution in the lowering operator $\hat{\sigma}_{-}$ [17], as it does not affect the calculation of either the emission spectrum or the intensity correlation discussed in Section 6. The emission spectrum of a single emitter is therefore equivalent for any detection point, except in the special case in which the detection direction is such that $\mathbf{e}_R$ and $\boldsymbol{\mu}$ are parallel, for which $\mathbf{E}_{\mu}(\mathbf{r}) = 0$ and no light is collected. Thus, we can directly identify $\hat{E}_{QE}^{(+)} = \hat{\sigma}_{-}$ and $\hat{E}_{QE}^{(-)} = \hat{\sigma}_{+}$ in the calculation of the emission spectrum (obtained then in arbitrary units), and compute the spectrum by adding Code 11 to the codes developed in the previous sections.

```
Rabi_freq = gamma_eg/100 #We consider weak driving
detuning = 0 # We consider again resonant driving
H = detuning * sigma_minus.dag() * sigma_minus - 0.5*Rabi_freq*(sigma_minus+sigma_minus.dag())
wlist = numpy.linspace(-5 * gamma_eg, 5 * gamma_eg, 500)
E_plus = sigma_minus
E_minus = sigma_minus.dag()
S = spectrum(H, wlist, c_ops, E_plus, E_minus)
#Plot
plt.figure(figsize=(4,4))
plt.plot(wlist / gamma_eg, S, lw=2)
plt.xlabel(r"$(\omega - \omega_L)/\gamma_{eg}$")
plt.ylabel(r"$S(\omega)$ [arb. u.]")
plt.show() #The resulting plot is shown in Fig. 5a
```

Code 11. Calculation of the steady-state emission spectrum of the emitter under resonant and weak driving. The function spectrum from QuTiP is used.

We plot in Fig. 5a the emission spectrum $S(\omega)$ obtained, under weak driving ($\Omega = \gamma_{eg}/100$) and resonant illumination. The spectrum exhibits a single peak at $\omega = \omega_L = \omega_{eg}$ with a full width at half maximum (FWHM) of γ_{eg} . This indicates that photons are emitted with the highest probability at the transition frequency of the emitter, corresponding to the relaxation of the excited state $|e\rangle$ into the ground state $|g\rangle$. The spectral linewidth is determined by the emitter lifetime, which can be understood from the time-frequency Heisenberg uncertainty principle. This behavior is consistent with the analytical results obtained using the Wigner-Weisskopf approximation in Section 3.2.b [see Eq. (11)].

The code can be easily adapted to explore the influence of different parameters on the emission spectrum. As an example, we compute the emission spectrum for different values of the Rabi frequency in Fig. 5b, namely $\Omega = \gamma_{eg}/100$ (solid blue line), $\Omega = 1.5\gamma_{eg}$ (solid orange line), and $\Omega = 4\gamma_{eg}$ (solid green line). We observe that, as the Rabi frequency increases, the emission spectrum develops a three-peak structure known as the Mollow triplet [40].

To better understand this phenomenon, it is convenient to treat the laser field quantum mechanically [20,41]. In this description, the laser field is modelled as a quantum harmonic oscillator of frequency ω_L and the emitter-field interaction is described by the Jaynes-Cummings Hamiltonian. The explicit form of this Hamiltonian and its diagonalization are presented in Section 7, where we discuss the analogous interaction between an emitter and an optical cavity. For the present discussion, it is sufficient to note that the eigenstates of the coupled system are superpositions of tensor-product states formed by the emitter states and the Fock states of the laser field. These eigenstates are organized into manifolds (or rungs), each consisting of a pair of dressed states. For example, the n -th manifold contains two states with a total of n excitations (between photons and emitter excitations), which can be written as

$$|\Omega_+^{(n)}\rangle = \sin \Theta_n |g\rangle|n\rangle + \cos \Theta_n |e\rangle|n-1\rangle, \quad |\Omega_-^{(n)}\rangle = \cos \Theta_n |g\rangle|n\rangle - \sin \Theta_n |e\rangle|n-1\rangle, \quad (23)$$

where $|n\rangle$ denotes the n -th Fock state of the electromagnetic mode associated with the laser field, and Θ_n is the mixing angle for the n -th manifold. For sufficiently large n (i.e., in the classical limit), $\tan(2\Theta_\infty) = |\Omega|/\Delta_{eg}$ and the energy splitting between $|\Omega_+^{(n)}\rangle$ and $|\Omega_-^{(n)}\rangle$ becomes $\hbar\sqrt{\Delta_{eg}^2 + |\Omega|^2}$ [49], as when the laser is treated classically and the rotating frame is considered (Section 4.a). Under resonant illumination, the energy splitting between $|\Omega_-^{(n)}\rangle$ and $|\Omega_+^{(n)}\rangle$ is $\hbar|\Omega|$ and the two states become nearly degenerate in the weak-driving regime. In addition, consecutive manifolds are separated in energy by $\hbar\omega_L$, corresponding to one additional photon in the laser field. Therefore, under weak driving, the only allowed one-photon transitions occur at the energy $\hbar\omega_L = \hbar\omega_{eg}$, in agreement with the simulations shown in Fig. 5a.

Figure 5c illustrates the energy-level structure of the n and $n+1$ manifolds for strong driving. In addition to transitions that emit photons at frequency $\omega = \omega_L = \omega_{eg}$ (black arrows), transitions emitting photons at frequencies $\omega = \omega_L - |\Omega|$ (red arrow) and $\omega = \omega_L + |\Omega|$ (purple arrow) also become allowed. The coexistence of these three possible transition frequencies explains the emergence of the Mollow triplet observed in Fig. 5b.

We suggest further adapting the code to calculate the emission spectrum in other situations, such as different laser frequencies (see Appendix A).

6. Intensity correlation and single photon emission

In the previous section, we discussed the characterization of the spectral properties of light through the emission spectrum. We now turn to the characterization of the statistical properties of light. To this end, we describe how to compute the intensity correlation function $g^{(2)}(\tau)$, also known as the second-order coherence function, using QuTiP.

$g^{(2)}(\tau)$ measures the probability of detecting two photons emitted by the system with a time delay τ , normalized by the probability of independent single-photon detection events [17,20,42,43]. Due to this normalization, $g^{(2)}(\tau)$ ranges from 0 to infinity. In particular, $g^{(2)}(\tau=0)$ is related to the probability of simultaneous emission of multiple photons or photon bunches. If $g^{(2)}(\tau=0) > g^{(2)}(\tau)$, the emitted light is said to be bunched, whereas if $g^{(2)}(\tau=0) < g^{(2)}(\tau)$ we say it is antibunched. Antibunching is a purely quantum phenomenon, as it reflects the tendency of the system to emit photons individually or in small groups. At the boundary between antibunched and bunched light lies coherent light (e.g., a laser), which satisfies $g^{(2)}(\tau) = 1$.

The intensity correlation function is again constructed from the positive- and negative-frequency electric field operators. Assuming, for simplicity, a scalar electric field operator, the second-order coherence function can be written as

$$g^{(2)}(\tau) = \frac{\langle \hat{E}^{(+)}(0)\hat{E}^{(+)}(\tau)\hat{E}^{(-)}(\tau)\hat{E}^{(-)}(0) \rangle_{ss}}{\langle \hat{E}^{(+)}(0)\hat{E}^{(-)}(0) \rangle_{ss}^2}, \quad (24)$$

where the numerator accounts for the probability of detecting two photons separated by a delay τ , while the denominator provides the normalization associated with single-photon detection probabilities. At sufficiently long delays, the photon detection events become uncorrelated and $g^{(2)}(\tau \rightarrow \infty) = 1$.

We next adapt the codes developed in previous sections to compute the intensity correlation function of light emitted by a single quantum emitter. We use the cyclic property of the trace, which allows us to rewrite the numerator in Eq. (24) as

$$\langle \hat{E}^{(+)}(0)\hat{E}^{(+)}(\tau)\hat{E}^{(-)}(\tau)\hat{E}^{(-)}(0) \rangle_{ss} = \text{Tr}[\hat{E}^{(+)}(0)\hat{E}^{(+)}(\tau)\hat{E}^{(-)}(\tau)\hat{E}^{(-)}(0)\hat{\rho}_{ss}] = \text{Tr}[I(t)\hat{\rho}_g], \quad (25)$$

where we have introduced $\hat{\rho}_g = \hat{E}^{(-)}(0)\hat{\rho}_{ss}\hat{E}^{(+)}(0)$ and the intensity operator $\hat{I}(t) = \hat{E}^{(+)}(t)\hat{E}^{(-)}(t)$.

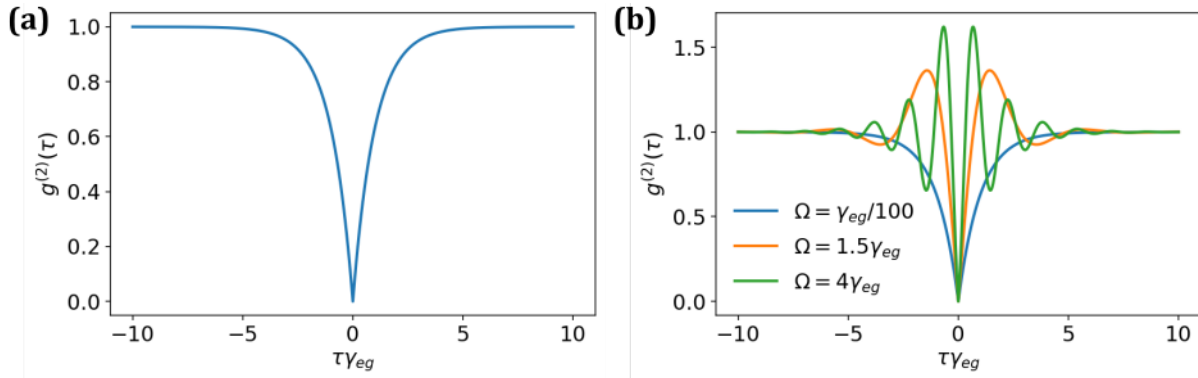


Fig. 6. Intensity correlation of light emitted from a quantum emitter. (a) Intensity correlation under weak driving ($\Omega = \gamma_{eg}/100$), showing a simple dip and $g^{(2)}(\tau = 0) = 0$. The simulation is performed using Code 12. (b) Intensity correlation for increasing values of the Rabi frequency: $\Omega = \gamma_{eg}/100$ (solid blue line), $\Omega = 1.5\gamma_{eg}$ (solid orange line), and $\Omega = 4\gamma_{eg}$ (solid green line). For larger values of the Rabi frequency, the intensity correlation exhibits Rabi oscillations. The transition frequency used in the simulations corresponds to a vacuum wavelength of 600 nm and the lifetime to $1/\gamma_{eg} = 10$ ns. The laser is resonantly tuned to the emitter transition frequency.

In this way, Eq. (25) can be calculated using mesolve with $\hat{\rho}_g$ as the initial state (Code 12). This is possible because mesolve can be used not only to compute the time evolution of the density matrix, but also to compute the expectation values of operators during the evolution. To do so, an additional input must be provided to mesolve, namely a list of target operators whose expectation values are to be evaluated during the time evolution.

```
Rabi_freq = gamma_eg/100 #We consider weak driving
detuning = 0 # We consider again resonant driving
H = detuning * sigma_minus.dag() * sigma_minus - 0.5*Rabi_freq*(sigma_minus+sigma_minus.dag())
rho_ss = steadystate(H, c_ops) #Calculation of the steady-state density matrix
rho_g = E_minus * rho_ss * E_plus #Calculation of rho_g in Eq. (25)
I_op = E_plus * E_minus #Intensity operator
taulist = numpy.linspace(0, 10, 400)/gamma_eg #Array with the delay times
result = mesolve(H, rho_g, taulist, c_ops, [I_op])
g2_numerator = result.expect[0] #Numerator of g2
g2_denominator = expect(I_op, rho_ss)**2 #Denominator of g2
g2 = g2_numerator / g2_denominator #Intensity correlation
tau_full = numpy.concatenate([-taulist[0:-1], taulist]) # Including negative delays
g2_full = numpy.concatenate((g2[0:-1], g2)) #Full g2 including negative delays
plt.figure(figsize=(6,4))
plt.plot(tau_full*gamma_eg, g2_full, lw=2)
plt.xlabel(r"$\tau \gamma_{eg}$")
plt.ylabel(r"$g^{(2)}(\tau)$")
plt.show() #The resulting plot is shown in Fig. 6a
```

Code 12. Calculation of the intensity correlation of light emitted from a quantum emitter. The result is plotted in Fig. 6a.

In Code 12 we use the symmetry relation $g^{(2)}(\tau) = g^{(2)}(-\tau)$, which allows us to obtain the standard representation of the intensity correlation function including negative delays. In addition, we again consider weak driving ($\Omega = \gamma_{eg}/100$) and resonant illumination. The plot is shown in Fig. 6a. We observe that the intensity correlation exhibits a pronounced dip at zero delay, with $g^{(2)}(\tau = 0) = 0$. This indicates that there is zero probability of simultaneously emitting more than one photon from a two-level quantum emitter. This behavior can be understood intuitively: the laser pumps the emitter into the excited state $|e\rangle$, which has a lifetime $1/\gamma_{eg}$, and this state then decays into the ground state $|g\rangle$, emitting a single photon. A second photon cannot be emitted until the emitter has relaxed and is subsequently re-excited.

Single-photon emission is one of the key properties of two-level quantum emitters, as it makes them promising photon sources for applications in quantum communication and quantum computing [1,2]. In

practice, the deviation of $g^{(2)}(\tau = 0)$ from zero is often used as a measure of the purity of single-photon emission in quantum emitters and other photon sources [8].

Moreover, Code 12 can be easily modified to investigate the behavior of the intensity correlation function when emitter or laser parameters are varied. As an example, we show in Fig. 6b the behavior of $g^{(2)}(\tau)$ for different values of the Rabi frequency, namely $\Omega = \gamma_{eg}/100$ (solid blue line), $\Omega = 1.5\gamma_{eg}$ (solid orange line), and $\Omega = 4\gamma_{eg}$ (solid green line). As the Rabi frequency increases, $g^{(2)}(\tau)$ continues to vanish at zero delay and, more interestingly, begins to exhibit Rabi oscillations. These oscillations arise from the emergence of dressed states, similarly to the oscillatory behavior observed in the population dynamics of the state $|e\rangle$ in Fig. 3. We analyse the case of off-resonant driving in Appendix A.

7. Interaction between a quantum emitter and an optical cavity

We have already explored the dynamics of a single quantum emitter and the spectral and statistical properties of the light it emits. In this section, we consider the situation in which the emitter interacts with an optical cavity mode. This allows us to introduce additional functionalities available in QuTiP, such as creation and annihilation operators and tensor products.

Specifically, we consider a cavity mode with frequency ω_c and decay rate κ . The Hamiltonian of this mode is that of a harmonic oscillator [31], $\hat{H}_c = \hbar \omega_c \hat{a}_c^\dagger \hat{a}_c$, where $\hat{a}_c^\dagger$ and $\hat{a}_c$ are the cavity creation and annihilation operators, respectively. These operators can be defined straightforwardly using the built-in QuTiP functions `create(N_c)` and `destroy(N_c)` from QuTiP (Code 13), where N_c is the finite dimension to which the Hilbert space of the harmonic oscillator is numerically truncated.

```
N_c = 5 #dimension at which we truncate the Hilbert space of the cavity mode
a_c = destroy(N_c)
ad_c = create(N_c)
```

Code 13. Definition of annihilation and creation operators using built-in functions from QuTiP.

Moreover, QuTiP provides a large collection of functions to facilitate work with cavity modes. For instance, Fock states $|n\rangle$ can be defined as `fock(N_c,n)`, while coherent states $|\alpha\rangle$ can be defined as `coherent(N_c,alpha)`, among many other available functions.

The interaction between the cavity mode and the emitter is typically described by the Jaynes-Cummings Hamiltonian, $\hat{H}_{JC} = \hbar g(\hat{a}_c^\dagger \hat{\sigma}_- + \hat{a}_c \hat{\sigma}_+)$, where g is the coupling strength. This Hamiltonian can again be obtained after applying the rotating-wave approximation (RWA) to the multipolar Hamiltonian, for which in this case the dipole moment operator of the emitter is proportional to $\hat{\sigma}_- + \hat{\sigma}_+$, and the electric field generated by the cavity mode is proportional to $\hat{a}_c^\dagger + \hat{a}_c$.

At this point, we note that when dealing with more than one system, every operator belongs to the global Hilbert space, given by the tensor product of the Hilbert spaces associated with the individual subsystems. Therefore, it is convenient to redefine all operators in the global Hilbert space of the emitter-cavity system. To this end, we note that the identity operator can be defined in QuTiP as `qeye(N)`, where N is the dimension of the corresponding Hilbert space. Additionally, the tensor product of two operators A and B can be obtained in QuTiP using `tensor(A,B)`. Therefore, the operators $\hat{\sigma}_-$, $\hat{\sigma}_+$, $\hat{a}_c$, and $\hat{a}_c^\dagger$ can be defined in the global Hilbert space as detailed in Code 14. A similar approach could be used, for example, to simulate the dynamics of several coupled quantum emitters [44,45].

```
sigma_minus_global = tensor(sigma_minus, qeye(N_c))
sigma_plus_global = tensor(sigma_minus.dag(), qeye(N_c))
a_c_global = tensor(qeye(2), a_c)
ad_c_global = tensor(qeye(2), ad_c)
```

Code 14. Definition of the emitter and cavity operators in the global Hilbert space.

In the absence of laser driving, the dynamics of the complete system is governed by the following master equation:

$$\frac{d}{dt}\hat{\rho}(t) = -\frac{i}{\hbar}[\hbar\omega_{eg}\hat{\sigma}_+\hat{\sigma}_- + \hbar\omega_c\hat{a}_c^\dagger\hat{a}_c + \hbar g(\hat{a}_c^\dagger\hat{\sigma}_- + \hat{a}_c\hat{\sigma}_+), \hat{\rho}(t)] + \frac{\gamma_{eg}}{2}\mathcal{D}[\hat{\sigma}_-]\hat{\rho}(t) + \frac{\kappa}{2}\mathcal{D}[\hat{a}_c]\hat{\rho}(t), \quad (26)$$

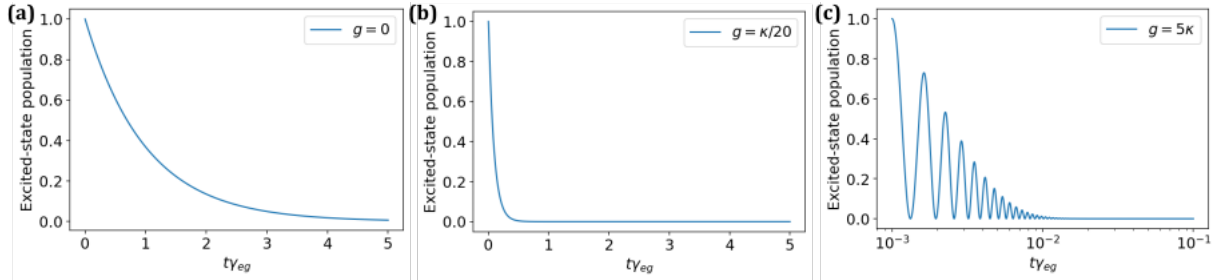


Fig. 7. Time evolution of the excited-state population of a quantum emitter in the absence and presence of an optical cavity. Codes 13–16 are used to solve the master equation (26). (a) The emitter is uncoupled from the cavity ($g = 0$). (b) The emitter is weakly coupled to the optical cavity ($g = \kappa/20$). (c) The emitter and the cavity are in the strong-coupling regime ($g = 5\kappa$). In all panels, the cavity mode is resonant with the emitter transition frequency ($\omega_c = \omega_{eg}$), the cavity decay rate is much larger than the emitter spontaneous emission rate ($\kappa = 1000\gamma_{eg}$), the emitter transition corresponds to a vacuum wavelength of 600 nm, and the emitter lifetime is $1/\gamma_{eg} = 10$ ns.

where κ is the dissipation rate of the cavity. In Code 15, we implement and solve the new master equation using `mesolve` once again. Specifically, we study the dynamics of the excited-state population of the emitter, which is initially prepared in its excited state with zero photons in the cavity. This builds upon all the concepts introduced so far. First, we consider a resonant cavity with a decay rate much larger than that of the emitter and operate in the weak-coupling regime ($g \ll \kappa$).

```
omega_c = omega_eg # Resonant cavity
kappa = gamma_eg*1000 # Cavity decay rate
g = kappa/20 #Weak coupling regime
H = (omega_eg * sigma_plus_global * sigma_minus_global + omega_c * ad_c_global * a_c_global + g *
(ad_c_global * sigma_minus_global + a_c_global * sigma_plus_global )) #new Hamiltonian
c_ops = [numpy.sqrt(gamma_eg) * sigma_minus_global, numpy.sqrt(kappa) * a_c_global]#new c_ops list
psi0 = tensor(ket_e, fock(N_c,0)) #We define the intial state using tensor and fock functions from QuTiP
tlist = numpy.linspace(0, 5/gamma_eg, 1000)
projector_e = sigma_plus_global * sigma_minus_global #Target operator for mesolve
result = mesolve(H, psi0, tlist, c_ops, [projector_e])
population_e = result.expect[0]
plt.figure(figsize=(6,4))
plt.plot(tlist * gamma_eg, population_e, label=r"$g=\kappa/20$")
plt.xlabel(r"$t \gamma_{eg}$")
plt.ylabel("Excited-state population")
plt.legend()
plt.show() #The resulting plot is shown in Fig. 7b
```

Code 15. Calculation of the dynamics of the emitter excited state when the system is weakly coupled to an optical cavity.

Figure 7b shows the excited-state dynamics obtained for $\kappa = 1000\gamma_{eg}$ and $g = \kappa/20$. For comparison, the same calculation is repeated for the uncoupled case by setting $g = 0$, and the result is shown in Fig. 7a. The comparison illustrates that coupling the emitter to the cavity accelerates its relaxation, leading to a significantly faster decay in Fig. 7b than in Fig. 7a. This phenomenon is known as the Purcell effect [46]. It arises from the modification of the local density of optical states produced by the cavity with respect to free space, which alters the spontaneous emission rate of the emitter. This effect can be understood analytically using the master-equation formalism [Eq. (12)] by tracing out the cavity Hilbert space (the environment) and reducing the description to the emitter Hilbert space alone (the system of interest). This treatment is valid as long as the emitter–cavity coupling remains weak and the cavity decay rate is much larger than that of the emitter. In this way, the effective decay rate of the emitter becomes [47,48]

$$\gamma_{eg}^{eff} = \gamma_{eg} + \kappa \frac{g^2}{(\kappa/2)^2 + (\omega_{eg} - \omega_c)^2}. \quad (27)$$

The Purcell factor is then defined as $F_p = \gamma_{eg}^{eff} / \gamma_{eg}$, which quantifies the enhancement of the emitter's

spontaneous emission rate due to its interaction with the cavity. The Purcell effect constitutes the foundation of many applications in nano-optics and cavity quantum electrodynamics.

Finally, we simulate the time evolution of the excited-state population for a larger emitter–cavity coupling strength ($g = 5\kappa$) and display the time axis on a logarithmic scale, as detailed in Code 16. The result is shown in Fig. 7c.

```
g = kappa*5 #Strong-coupling regime
H = (omega_eg * sigma_plus_global * sigma_minus_global + omega_c * ad_c_global * a_c_global + g *
(ad_c_global * sigma_minus_global + a_c_global * sigma_plus_global )) #new Hamiltonian
tlist = numpy.logspace(-3, 1, 10000)/gamma_eg
result = mesolve(H, psi0, tlist, c_ops, [projector_e])
population_e = result.expect[0]
plt.figure(figsize=(6,4))
plt.plot(tlist * gamma_eg, population_e, label=r"$g=5\kappa$")
plt.xlabel(r"$t \backslash \gamma_{eg}$")
plt.ylabel("Excited-state population")
plt.xscale("log")
plt.legend()
plt.show() #The resulting plot is shown in Fig. 7c
```

Code 16. Calculation of the dynamics of the emitter excited state when the system is strongly coupled to an optical cavity.

In comparison to the weak-coupling regime (Fig. 7b), the decay of the excited-state population in Fig. 7c is now much faster and, crucially, it is accompanied by oscillations. These oscillations are a signature of the hybridization between the emitter and the cavity mode. To better understand this hybridization, we diagonalize the full Hamiltonian $\hat{H}_{QE} + \hat{H}_C + \hat{H}_{JC}$. The eigenstates are organized into an infinite ladder of excitation manifolds. The n -th manifold contains two eigenstates,

$$|E_+^{(n)}\rangle = \sin \Theta_n |g\rangle|n\rangle + \cos \Theta_n |e\rangle|n-1\rangle, \quad |E_-^{(n)}\rangle = \cos \Theta_n |g\rangle|n\rangle - \sin \Theta_n |e\rangle|n-1\rangle, \quad (28)$$

where the mixing angle satisfies $\tan(2\Theta_n) = 2|g|\sqrt{n}/(\omega_{eg} - \omega_c)$. The corresponding eigenenergies are

$$E_+^{(n)} = \hbar \left[n\omega_c + \frac{\omega_{eg} - \omega_c}{2} \right] + \frac{\hbar}{2} \sqrt{(\omega_{eg} - \omega_c)^2 + 4g^2n},$$

$$E_-^{(n)} = \hbar \left[n\omega_c + \frac{\omega_{eg} - \omega_c}{2} \right] - \frac{\hbar}{2} \sqrt{(\omega_{eg} - \omega_c)^2 + 4g^2n}, \quad (29)$$

so that their energy splitting is $\hbar\sqrt{(\omega_{eg} - \omega_c)^2 + 4g^2n}$. We note that the states $|E_+^{(n)}\rangle$ and $|E_-^{(n)}\rangle$ in Eq. (28) are formally identical to the dressed states $|\Omega_+^{(n)}\rangle$ and $|\Omega_-^{(n)}\rangle$ in Eq. (23), except that the Fock state now describes the cavity mode rather than the driving laser field. Taking the classical limit ($n \gg 1$) in Eq. (29), $2|g|\sqrt{n}$ can be identified with the Rabi frequency of the classical drive [49], thereby recovering the results discussed in Section 5.

To analyze the dynamics shown in Fig. 7c, it is sufficient to restrict the discussion to the single-excitation manifold ($n = 1$). Since the initial state contains a single excitation, the Jaynes–Cummings Hamiltonian conserves the total number of excitations, and no external driving field is present, the system evolves entirely within this manifold. Further, because the cavity is resonant with the emitter, the energy splitting between $|E_+^{(1)}\rangle$ and $|E_-^{(1)}\rangle$ reduces to $2|g|$, which is known as the vacuum Rabi splitting. When this splitting becomes comparable to or larger than the linewidths of the emitter and the cavity, the system enters the strong-coupling regime. In this regime, the coherent exchange of excitation between the emitter and the cavity gives rise to the oscillations observed in Fig. 7c. This behavior is analogous to the Rabi oscillations discussed in Fig. 3b for an emitter strongly driven by a classical laser. In both cases, the excited state of the emitter is no longer an eigenstate of the coupled system. Instead, it is a coherent superposition of the true

eigenstates, whose different eigenenergies produce oscillations through quantum interference during the time evolution. $|E_+^{(1)}\rangle$ and $|E_-^{(1)}\rangle$ are hybrid light-matter states, commonly interpreted as quasiparticles known as the upper and lower polaritons [50,51], respectively. The study of polaritonic systems has become a highly active area of research owing to their promising applications in chemistry, nanophotonics, cavity quantum electrodynamics, and emerging quantum technologies.

Lastly, we emphasize that if the coupling strength becomes comparable to the cavity and emitter frequencies, the system enters the so-called ultrastrong-coupling regime [52], which requires a more sophisticated description than that provided by the master equation (26), a topic that lies beyond the scope of this tutorial.

8. Conclusions

We have presented an introduction to light-matter interaction in two-level quantum emitters for undergraduate students, based on numerical simulations performed with QuTiP. The codes and simulations are accompanied by intuitive explanations and concise analytical derivations aimed at highlighting the essential physical concepts. Importantly, the provided codes can be readily adapted to systematically explore the effect of different physical parameters, helping students develop a deeper physical intuition while becoming familiar with the use of QuTiP. We therefore expect this tutorial to serve as an accessible introduction to quantum optics, as well as a practical starting point for the numerical modelling of quantum emitters and nanophotonic systems.

Acknowledgements

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Appendix A. Off-resonant laser driving of a quantum emitter

In Sections 4 to 6, we focused on analyzing the dynamics and the light emission of a quantum emitter driven exactly on resonance. Throughout these sections, we encouraged the reader to perform additional simulations to become familiar with QuTiP and to explore the influence of different physical parameters. In this Appendix, we discuss the dynamics of the emitter and the properties of its emission when the driving laser is slightly detuned from the emitter transition.

We first analyze the evolution of the excited-state population in the rotating frame at the laser frequency, assuming that the emitter is initially prepared in the excited state, as in Fig. 3. The complete code is provided in Code 17, and the corresponding results are shown in Fig. 8a. The Rabi frequency is fixed at $\Omega = 5\gamma_{eg}$, and three different values of laser detuning $\Delta_{eg} = \omega_{eg} - \omega_L$ are considered. Specifically, we examine the cases of exact resonance $\Delta_{eg} = 0$ (blue line in Fig. 8a), $\Delta_{eg} = 2\gamma_{eg}$ (orange line) and $\Delta_{eg} = -4\gamma_{eg}$ (green line). The driving strength is sufficiently large to induce Rabi oscillations. We observe that the period of these oscillations decreases as the magnitude of the laser detuning increases. As discussed in Section 5, the dressed states are separated in energy by $\hbar\Omega_R = \hbar\sqrt{\Delta_{eg}^2 + |\Omega|^2}$ in the rotating frame, with Ω_R called generalized Rabi frequency. Consequently, the oscillation frequency increases with the absolute value of the laser detuning, leading to a shorter oscillation period. Further, Fig. 8a shows that the steady-state population of the excited state decreases as $|\Delta_{eg}|$ increases. The reason for this is that for larger laser detunings, the efficiency of exciting the emitter decreases. Thus, the threshold of driving strength to reach the emitter saturation increases, consistently with the results in Fig. 4b.

We next analyze the emission spectrum for the same driving strength and the same three values of the laser detuning (Code 18). The resulting spectra are shown in Fig. 8b. For $\Delta_{eg} = 0$ (blue line), we recover the Mollow triplet discussed in Section 5, consisting of a dominant central peak and two symmetric sidebands separated from the central peak by $|\Omega|$. For off-resonant driving, we observe that the separation between the central peak and the sidebands increases owing to the increase in the generalized Rabi frequency Ω_R . Interestingly, the two sidebands become asymmetric, exhibiting different intensities and linewidths [53,54]. This behaviour can be understood in terms of the dressed-state picture. Under resonant driving, $\tan(2\theta_\infty) = |\Omega|/\Delta_{eg} \rightarrow \infty$, so that, in the limit of classical laser driving, the dressed states become exact symmetric and antisymmetric superpositions of the bare ground and excited states. Specifically,

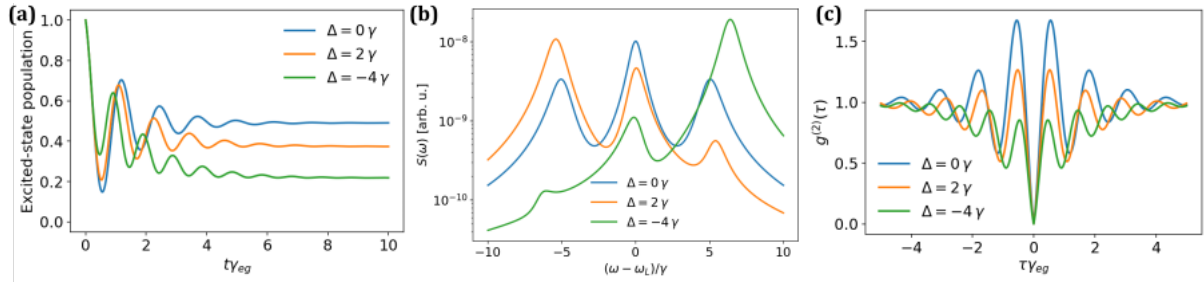


Fig. 8. Excited-state dynamics and emission properties of an off-resonantly driven quantum emitter. (a) Time evolution of the excited-state population. Code 17 is used. (b) Emission spectrum. Code 18 is used. (c) Intensity correlation. Code 19 is used. The blue lines correspond to resonant driving ($\Delta_{eg} = 0$), the orange lines to a detuning of $\Delta_{eg} = 2\gamma_{eg}$, and the green lines to $\Delta_{eg} = -4\gamma_{eg}$. The Rabi frequency is fixed at $\Omega = 5\gamma_{eg}$, the emitter transition corresponds to a vacuum wavelength of 600 nm, and the emitter lifetime is $1/\gamma_{eg} = 10$ ns.

```
import numpy
from qutip import *
import matplotlib.pyplot as plt
sigma_minus = sigmam()
c = 3e8
wavelength = 600e-9
omega_eg = 2 * numpy.pi * c / wavelength
gamma_eg = 1 / (10e-9)
Rabi_freq = 5 * gamma_eg
omega_L_list = [omega_eg, omega_eg - 2*gamma_eg, omega_eg + 4*gamma_eg]
plt.figure(figsize=(6, 4))
for omega_L in omega_L_list:
    detuning = omega_eg - omega_L
    H = detuning * sigma_minus.dag() * sigma_minus - 0.5*Rabi_freq*(sigma_minus + sigma_minus.dag())
    c_ops = [numpy.sqrt(gamma_eg) * sigma_minus]
    tlist = numpy.linspace(0, 10, 300)/gamma_eg
    rho0 = sigma_minus.dag() * sigma_minus
    result = mesolve(H, rho0, tlist, c_ops)
    population_e = []
    for rho in result.states:
        population_e.append(rho[0,0].real)
    plt.plot(tlist*gamma_eg, population_e, lw=2, label=fr"\Delta = {detuning/gamma_eg:.0f}\,\,\gamma_{eg}")
plt.xlabel(r"$t\gamma_{eg}$")
plt.ylabel("Excited-state population")
plt.ylim(-0.05, 1.05)
plt.legend(frameon=False)
```

Code 17. Simulation of the time evolution of the excited-state population of the driven emitter for different values of laser frequency.

$|\Omega_+^{(n)}\rangle = (|g\rangle|n\rangle + |e\rangle|n-1\rangle)/\sqrt{2}$ and $|\Omega_-^{(n)}\rangle = (|g\rangle|n\rangle - |e\rangle|n-1\rangle)/\sqrt{2}$. Consequently, the one-photon transitions indicated by the red and purple arrows in Fig. 5c, which give rise to the two sidebands in the spectrum, have identical transition probabilities, leading to a symmetric Mollow triplet. In contrast, when $\Delta_{eg} \neq 0$, the dressed states $|\Omega_+^{(n)}\rangle$ and $|\Omega_-^{(n)}\rangle$ are no longer exact symmetric and antisymmetric superpositions of the bare states. As a result, the transition probabilities associated with the red and purple arrows in Fig. 5c become different, giving rise to the asymmetric sidebands observed in Fig. 8b for $\Delta_{eg} = 2\gamma_{eg}$ (orange line) and $\Delta_{eg} = -4\gamma_{eg}$ (green line). Moreover, the sign of the asymmetry depends on the sign of the laser detuning, just as $\tan(2\Theta_\infty) = |\Omega|/\Delta_{eg}$ does.

```
wlist = numpy.linspace(-10 * gamma_eg, 10 * gamma_eg, 500)
plt.figure(figsize=(7,5))
for omega_L in omega_L_list:
    detuning = omega_eg-omega_L
    H = detuning * sigma_minus.dag() * sigma_minus - 0.5 * Rabi_freq * (sigma_minus +
sigma_minus.dag())
    rho_ss = steadystate(H, c_ops) # Steady-state density matrix
    E_plus = sigma_minus
    E_minus = sigma_minus.dag()
    S = spectrum(H, wlist, c_ops, E_plus, E_minus)
    plt.plot(wlist / gamma_eg, S, lw=2, label=fr"$\Delta = \{{detuning/gamma_eg:.0f}\}\backslash, \backslash gamma$")
plt.xlabel(r"$\{{\omega - \omega_L}\} / \backslash gamma$")
plt.ylabel(r"$S(\backslash omega)$ [arb. u.]")
plt.yscale("log")
plt.legend(frameon=False)
plt.show() #The resulting plot is shown in Fig. 8b
```

Code 18. Simulation of the emission spectrum of the driven emitter for different values of laser frequency.

Finally, we analyze the behaviour of the intensity correlation $g^{(2)}(\tau)$ for the same values of the laser detuning and again with the Rabi frequency fixed at $\Omega = 5\gamma_{eg}$ (Code 19). The results are shown in Fig. 8c. We observe a behaviour similar to that discussed in Section 6, with $g^{(2)}(\tau = 0) = 0$, reflecting the perfect single-photon emission of the two-level emitter. The main difference is the decrease in the period of the oscillations as the magnitude of the laser detuning increases, which results from the increase in the generalized Rabi frequency.

```
taulist = numpy.linspace(0, 5, 400)/gamma_eg #Array with the delay times
plt.figure(figsize=(6,4))
E_plus = sigma_minus
E_minus = sigma_minus.dag()
for omega_L in omega_L_list:
    detuning = omega_eg-omega_L
    H = detuning * sigma_minus.dag() * sigma_minus - 0.5 * Rabi_freq * (sigma_minus +
sigma_minus.dag())
    rho_ss = steadystate(H, c_ops) #Calculation of the steady-state density matrix
    rho_g = E_minus * rho_ss * E_plus #Calculation of rho_g in Eq. (24)
    I_op = E_plus * E_minus #Intensity operator
    result = mesolve(H, rho_g, taulist, c_ops, [I_op])
    g2_numerator = result.expect[0] #Numerator of g2
    g2_denominator = expect(I_op, rho_ss)**2 #Denominator of g2
    g2 = g2_numerator / g2_denominator #Intensity correlation
    tau_full = numpy.concatenate([-taulist[:0:-1], taulist]) # Including negative delays
    g2_full = numpy.concatenate([g2[:0:-1], g2]) #Full g2 including negative delays
    plt.plot(tau_full*gamma_eg, g2_full, lw=2, label=fr"$\Delta = \{{detuning/gamma_eg:.0f}\}\backslash, \backslash gamma$")
plt.xlabel(r"$\tau \backslash gamma_{eg}$")
plt.ylabel(r"$g^{\{{2\}}(\tau)$")
plt.legend(frameon=False)
plt.show() #The resulting plot is shown in Fig. 8c
```

Code 19. Simulation of the intensity correlation from the driven emitter for different values of laser frequency.

Appendix B. Incoherent pumping of a quantum emitter.

In the main text, we have focused on the case of coherent laser driving of the emitter. For completeness, we discuss in this appendix the case of incoherent pumping. Within the master-equation formalism, incoherent pumping is incorporated by including a Lindblad dissipator [Eq. (14)] with a Lindblad operator corresponding to a raising (creation) operator. The associated rate, P , therefore represents an incoherent pumping rate rather than a decay rate.

Incoherent pumping can be used to describe excitation by a thermal environment. In this case, the incoherent pumping rate is proportional to the thermal occupation of the environment, n_{th} , and to the spontaneous emission rate γ [27]. Concretely, $P = \gamma n_{th}$. The thermal environment also affects the emission rate of the emitter, as spontaneous emission (with rate γ) is also accompanied by stimulated emission (with rate γn_{th}). Thus, the total emission rate can be defined as $\gamma_{eg} = \gamma(1 + n_{th})$. For optical transition frequencies, however, the thermal occupation is negligible even at room temperature, which is why incoherent pumping has been neglected throughout the main text.

Incoherent pumping can also arise from the coherent excitation of higher-energy states that are omitted from the effective model because they rapidly decay into an excited state of the system [55,56]. The resulting population transfer is then described as an effective incoherent pumping process. For a molecular emitter, for example, these higher-energy states may correspond to vibrationally excited levels within the electronically excited manifold, which relax to the vibrational ground level of the same electronic manifold much faster than the electronic relaxation. Typically, vibrational relaxation in organic molecules occurs on the picosecond timescale, whereas electronic relaxation takes place on the nanosecond timescale [29,57]. When incoherent pumping results from the excitation of a rapidly decaying higher-energy state, the effective pumping rate P can be understood as being proportional to the intensity of the driving field and to the lifetime of the rapidly decaying higher-energy state [55,56].

We next discuss the dynamics and emission properties of an incoherently pumped emitter. To this end, we again consider the master equation (13), which does not include coherent laser excitation and incorporates an incoherent-pumping Lindblad dissipator. Additionally, we move to a rotating frame at frequency ω_{eg} , which facilitates the numerical simulations and the analytical derivations. The dynamics of the incoherently pumped emitter is then governed by

$$\frac{d}{dt} \hat{\rho}_{QE}(t) = \frac{\gamma_{eg}}{2} \mathcal{D}[\hat{\sigma}_-] \hat{\rho}_{QE}(t) + \frac{P}{2} \mathcal{D}[\hat{\sigma}_+] \hat{\rho}_{QE}(t). \quad (30)$$

We first analyze the dynamics of the excited-state population under incoherent pumping. Code 20 provides the numerical implementation used to solve the time evolution of the excited-state population for an initially excited emitter, and the corresponding results are shown in Fig. 9a. Specifically, we consider three pumping rates: $P = \gamma_{eg}/10$ (blue line in Fig. 9a), $P = \gamma_{eg}$ (orange line), and $P = 10\gamma_{eg}$ (green line).

For $P = \gamma_{eg}/10$, the excited-state population exhibits the same qualitative behavior as in the weak coherent-driving regime, namely a simple exponential decay towards an almost vanishing steady-state population. However, significant differences emerge for larger incoherent pumping rates. We find that for increasing values of P , the excited-state population reaches its steady value faster. To gain analytical insight into this behavior, we derive the equation of motion of the excited-state population $p_e(t) = \langle e | \hat{\rho}_{QE}(t) | e \rangle$ from Eq. (30):

$$\frac{d}{dt} p_e(t) = -\gamma_{eg} p_e(t) + P p_g(t) = -(\gamma_{eg} + P) p_e(t) + P, \quad (31)$$

where $p_g(t) = \langle g | \hat{\rho}_{QE}(t) | g \rangle$ represents the ground-state population. Equation (31) reveals that increasing P effectively decreases the relaxation time.

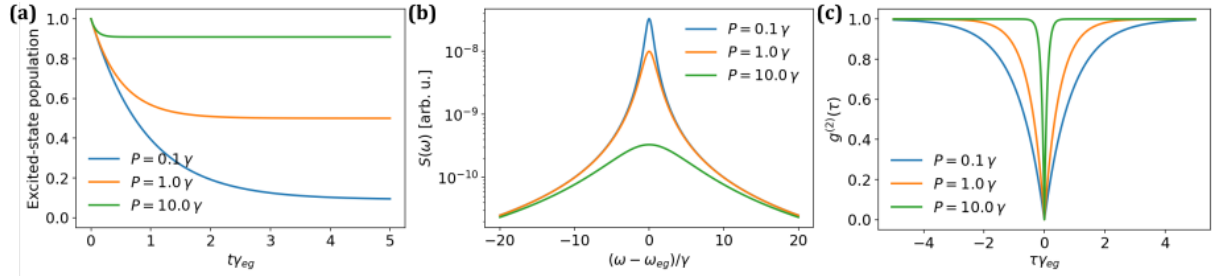


Fig. 9. Excited-state dynamics and emission properties of an incoherently pumped quantum emitter. (a) Time evolution of the excited-state population. Code 20 is used. (b) Emission spectrum. Code 21 is used. (c) Intensity correlation. Code 22 is used. The blue lines correspond to $P = \gamma_{eg}/10$, the orange lines to $P = \gamma_{eg}$, and the green lines to $P = 10\gamma_{eg}$. The emitter transition corresponds to a vacuum wavelength of 600 nm, and the emitter lifetime is $1/\gamma_{eg} = 10$ ns.

```
import numpy
from qutip import *
sigma_minus = sigmam()
c = 3e8
wavelength = 600e-9
gamma_eg = 1 / (10e-9)
H = 0 * sigma_minus.dag() * sigma_minus
P_list = [gamma_eg/10, gamma_eg, gamma_eg*10]
plt.figure(figsize=(6, 4))
for P in P_list:
    c_ops = [numpy.sqrt(gamma_eg) * sigma_minus, numpy.sqrt(P) * sigma_minus.dag()]
    tlist = numpy.linspace(0, 5, 300)/gamma_eg
    rho0 = sigma_minus.dag() * sigma_minus
    result = mesolve(H, rho0, tlist, c_ops)
    population_e = []
    for rho in result.states:
        population_e.append(rho[0,0].real)
    plt.plot(tlist*gamma_eg, population_e, lw=2, label=fr"$P = \{P/\gamma_{eg}:1f\} \backslash \gamma_{eg}$")
plt.xlabel(r"$t \backslash \gamma_{eg}$")
plt.ylabel("Excited-state population")
plt.ylim(-0.05, 1.05)
plt.legend(frameon=False)
plt.show() #The resulting plot is shown in Fig. 9a
```

Code 20. Simulation of the time evolution of the excited-state population of the incoherently pumped emitter.

Further, the steady excited-state population increases with P . Interestingly, for $P = 10\gamma_{eg}$, the steady-state excited-state population exceeds 0.5, demonstrating that an incoherently pumped two-level emitter is not necessarily limited by the saturation value characteristic of coherent driving. This behavior can be understood analytically by imposing the steady-state condition in Eq. (31), $\frac{d}{dt}\hat{\rho}_{QE}(t) = 0$. One then obtains $p_e(t \rightarrow \infty) = P/(\gamma_{eg} + P)$, which shows that the excited-state population approaches unity in the limit $P \gg \gamma_{eg}$. This result applies when the incoherent pumping originates from the excitation of rapidly decaying higher-energy states. It does not, however, apply to thermal pumping, for which P can never exceed γ_{eg} , even in the limit of very large n_{th} , since $\gamma_{eg} = \gamma(1 + n_{th})$ and $P = \gamma n_{th}$. As a result, $p_e(t \rightarrow \infty)$ cannot exceed 0.5 under thermal incoherent pumping (see orange line in Fig. 9c, for which $P = \gamma_{eg}$), just as under coherent driving.

Moreover, Fig. 9a shows that the excited-state population decays without exhibiting Rabi oscillations. This is because incoherent pumping does not modify the eigenstates of the system, unlike coherent driving, and therefore does not induce coherent population oscillations.

We next compute the emission spectrum for the same three different values of P using Code 21. The results are plotted in Fig. 9b. We find that a single peak emerges even at high P , which is again due to the fact that the incoherent pumping does not dress the eigenstates of the system, preventing the emergence of the Mollow triplet. However, what we observe is that the peak broadens as P increases, which is due to the increase of the effective relaxation rate.

```
wlist = numpy.linspace(-20 * gamma_eg, 20 * gamma_eg, 500)
plt.figure(figsize=(6,4))
for P in P_list:
    c_ops = [numpy.sqrt(gamma_eg) * sigma_minus, numpy.sqrt(P) * sigma_minus.dag()]
    rho_ss = steadystate(H, c_ops) # Steady-state density matrix
    E_plus = sigma_minus
    E_minus = sigma_minus.dag()
    S = spectrum(H, wlist, c_ops, E_plus, E_minus)
    plt.plot(wlist/ gamma_eg, S, lw=2, label=fr"$P = \{P/\gamma_{eg}:.1f\} \backslash \gamma_{eg}$")
plt.xlabel(r"$\omega - \omega_{eg} / \gamma_{eg}$")
plt.ylabel(r"$S(\omega)$ [arb. u.]")
plt.yscale("log")
plt.legend(frameon=False)
plt.show() #The resulting plot is shown in Fig. 9b
```

Code 21. Simulation of the emission spectrum of the incoherently pumped emitter.

Finally, we compute the intensity correlation $g^{(2)}(\tau)$ using Code 22. The results, shown in Fig. 9c, reveal again the ideal single-photon emission from the two-level emitter, as $g^{(2)}(0) = 0$ independently of the value of P . Additionally, the width of the dip narrows, again due to the increase of the effective relaxation rate, and Rabi oscillations do not emerge, as the eigenstates are not dressed.

```
taulist = numpy.linspace(0, 5, 400)/gamma_eg #Array with the delay times
plt.figure(figsize=(6,4))
E_plus = sigma_minus
E_minus = sigma_minus.dag()
for P in P_list:
    c_ops = [numpy.sqrt(gamma_eg) * sigma_minus, numpy.sqrt(P) * sigma_minus.dag()]
    rho_ss = steadystate(H, c_ops) #Calculation of the steady-state density matrix
    rho_g = E_minus * rho_ss * E_plus #Calculation of rho_g in Eq. (24)
    I_op = E_plus * E_minus #Intensity operator
    result = mesolve(H, rho_g, taulist, c_ops, [I_op])
    g2_numerator = result.expect[0] #Numerator of g2
    g2_denominator = expect(I_op, rho_ss)**2 #Denominator of g2
    g2 = g2_numerator / g2_denominator #Intensity correlation
    tau_full = numpy.concatenate((-taulist[:0:-1], taulist)) # Including negative delays
    g2_full = numpy.concatenate((g2[:0:-1], g2)) #Full g2 including negative delays
    plt.plot(tau_full*gamma_eg, g2_full, lw=2, label=fr"$P = \{P/\gamma_{eg}:.1f\} \backslash \gamma_{eg}$")
plt.xlabel(r"$\tau \gamma_{eg}$")
plt.ylabel(r"$g^{(2)}(\tau)$")
plt.legend(frameon=False)
plt.show() #The resulting plot is shown in Fig. 9c
```

Code 22. Simulation of the intensity correlation from the incoherently pumped emitter.