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Model of point source for layered metamaterials

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ABSTRACT:

The multilayer system including negative index material (NIM) layers is examined. We deal with the NIM system composed of arbitrary finite number of parallel alternated layers filled with isotropic homogeneous NIM and vacuum. The Maxwell's equations for the point source are considered. The NIM layer has the electric permittivity and magnetic permeability, which are equal to −1 for the certain frequency (NIM frequency). We set the goal of obtaining expressions for the electric Green's function. The Laplace and Fourier transforms are used. The differential equations for the scalar *s*- and *p*-polarization parts of the electric Green's function are obtained. The solutions of the differential equations are obtained in the travelling wave form with unknown coefficients. With the standard boundary conditions for every layer, the recurrence relations for the coefficients are obtained. The solution is obtained by the generating function method. The expressions for the scalar *s*- and *p*-polarization and vector part of the electric Green's function are derived. Under some assumptions, we observe the reflection absence (for the main term of the solution asymptotics near the NIM frequency). The obtained results can be used in simulation or engineering of real objects, such as superlens systems and multilayer NIM coverings.

Key words: metamaterials, negative index material, Maxwell's equations, Green's function

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1. Introduction

Metamaterials (negative index materials, NIM) are artificial materials, the main feature of which is the negative refractive index. By using NIMs, new covered surfaces, cloaking and invisible materials can be created [1, 2], as well as superlenses with the resolving power, many times exceeding the diffraction limit [3-5]. In the general case, NIMs are characterized by an occurrence of the ω frequencies wherewith the electric $\varepsilon(\omega)$ permittivity and $\mu(\omega)$ magnetic permeability (and, consequently, the refractive index) possess negative values [6, 7]. In the particular case [8] called NIM situation, with the $\hat{\omega}$ frequency called NIM frequency, these $\varepsilon(\omega)$ and $\mu(\omega)$ are equal to –1 (which is opposed to the vacuum value of +1). Systems with NIM elements are called NIM systems.

Among NIM systems, layered NIM systems are widely known. The simplest model of the layered NIM system (which is a two-layer system) was considered in [8, 9]. The three-layer NIM system, which is the model for the superlens, was studied in [3, 4, 5, 10, 11]. The investigations of the multilayer NIM systems (i.e., systems with the count of layers greater than three) are presented in [12-15].

A number of NIM studies are dedicated to obtaining the Green's function which describes an electromagnetic field of a point source. An electric field value can be determined by using of the Green's function. In [8] a two-layer NIM system composed of homogeneous isotropic half-spaces filled with NIM and vacuum, was in detail studied. The expressions for the scalar *s*- and *p*-polarization parts of the electric Green's function were obtained in the NIM situation. In [11] the three-layer isotropic NIM system was examined.

The goal of our work is to obtain expressions for the electric Green's function in the NIM situation for a multilayer NIM system, composed of arbitrary finite number of parallel layers. The layers are filled with NIM and vacuum, and disposed alternately. Taking into account boundary conditions at layer's surfaces, we use the recurrence relation method. The method is easy to employ because of the solution interdependence in adjoining layers, and appropriate to analyze and compare the solutions obtained.

2. Green's function approach

2.a. Maxwell's equations

Maxwell's equations describe the electromagnetic field, and, for the case that permanent polarization and magnetization are absent, are presented in a differential form as follows (we set $\varepsilon_0 = \mu_0 = 1$ for brevity [8]):

$$\frac{d\mathbf{D}}{dt}(\mathbf{x},t) = \nabla \times \mathbf{H}(\mathbf{x},t)\,, \tag{1}$$

$$\frac{d\mathbf{B}}{dt}(\mathbf{x},t) = -\nabla \times \mathbf{E}(\mathbf{x},t)\,, \tag{2}$$

$$\nabla \cdot \mathbf{D}(\mathbf{x},t) = 0\,, \tag{3}$$

$$\nabla \cdot \mathbf{B}(\mathbf{x},t) = 0\,, \tag{4}$$

where the $\mathbf{x}$ vector is located in the $\{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}$ basis of the Cartesian coordinates, ∇ is the Hamilton operator, $\times$ is a cross product symbol, $\cdot$ is an inner product symbol as well as a symbol for the matrix product. A medium reaction to the electromagnetic field is described with the following auxiliary field equations:

$$\mathbf{D}(\mathbf{x},t) = \mathbf{E}(\mathbf{x},t) + \mathbf{P}(\mathbf{x},t),\ \mathbf{P}(\mathbf{x},t) = \int_{t_0}^{t} \boldsymbol{\chi}_e(\mathbf{x},t-s)\cdot \mathbf{E}(\mathbf{x},s)\,ds\,, \tag{5}$$

$$\mathbf{B}(\mathbf{x},t) = \mathbf{H}(\mathbf{x},t) + \mathbf{M}(\mathbf{x},t)\,,\ \mathbf{M}(\mathbf{x},t) = \int_{t_0}^{t} \boldsymbol{\chi}_m(\mathbf{x},t-s)\cdot \mathbf{H}(\mathbf{x},s)\,ds\,, \tag{6}$$

where $\boldsymbol{\chi}_e(\mathbf{x},t)$, $\boldsymbol{\chi}_m(\mathbf{x},t)$ are the electric and magnetic susceptibility tensors. According to the causality condition, one has

$$\boldsymbol{\chi}_e(\mathbf{x},t) = \boldsymbol{\chi}_m(\mathbf{x},t) = 0 \text{ for } t < t_0. \tag{7}$$

Here t_0 is the initial time. It means that the polarization and magnetization vanish for times smaller than some finite t_0, which can have any value [8]. In relation to the Laplace transform

$$\hat{f}(z) = \int_0^{\infty} e^{izt} f(t)dt\,,\ f(t) = \frac{1}{2\pi}\int_{\Gamma} e^{-izt}\hat{f}(z)\,dz\,, \tag{8}$$

where $z = \omega + i\alpha$, $\alpha \to 0$, $\alpha > 0$, and Γ is a straight line located parallel at an α range to the real axis, it is proper to consider that $t \geq 0$, therefore we assume $t_0 = 0$. We also assume that the NIM system is passive [8]. Then, the electromagnetic energy

$$U_{em}(t) = \frac{1}{2}\int\left[\mathbf{E}^2(\mathbf{x},t) + \mathbf{H}^2(\mathbf{x},t)\right]d\mathbf{x} \tag{9}$$

is a non-increasing function of time. With the causality and passivity conditions and AFF (auxiliary field formalism), the NIM system has a proper time evolution [8]. In case the initial fields are square integrable they remain so for all later times.

Before introducing the Green's function, we apply the Laplace transform to the Maxwell's equations (1)-(4) [8]:

$$-iz\hat{\mathbf{D}}(\mathbf{x},z) - \mathbf{D}(\mathbf{x},0) = \nabla \times \hat{\mathbf{H}}(\mathbf{x},z)\,, \tag{10}$$

$$-iz\hat{\mathbf{B}}(\mathbf{x},z) - \mathbf{B}(\mathbf{x},0) = -\nabla \times \hat{\mathbf{E}}(\mathbf{x},z)\,, \tag{11}$$

$$\hat{\mathbf{D}}(\mathbf{x},z) = \boldsymbol{\varepsilon}(\mathbf{x},z)\cdot \hat{\mathbf{E}}(\mathbf{x},z)\,, \tag{12}$$

$$\hat{\mathbf{B}}(\mathbf{x},z) = \boldsymbol{\mu}(\mathbf{x},z)\cdot \hat{\mathbf{H}}(\mathbf{x},z)\,, \tag{13}$$

where

$$\boldsymbol{\varepsilon}(\mathbf{x},z)=1+\hat{\chi}_e(\mathbf{x},z), \tag{14}$$

$$\boldsymbol{\mu}(\mathbf{x},z)=1+\hat{\chi}_m(\mathbf{x},z). \tag{15}$$

According to the causality condition (7), we obtain the following relations from the auxiliary equations (5) and (6):

$$\mathbf{D}(\mathbf{x},0)=\mathbf{E}(\mathbf{x},0), \tag{16}$$

$$\mathbf{B}(\mathbf{x},0)=\mathbf{H}(\mathbf{x},0). \tag{17}$$

Then, equations (10) and (11) acquire a form:

$$-iz\boldsymbol{\varepsilon}(\mathbf{x},z)\cdot\hat{\mathbf{E}}(\mathbf{x},z)-\mathbf{E}(\mathbf{x},0)=\nabla\times\hat{\mathbf{H}}(\mathbf{x},z), \tag{18}$$

$$-iz\boldsymbol{\mu}(\mathbf{x},z)\cdot\hat{\mathbf{H}}(\mathbf{x},z)-\mathbf{H}(\mathbf{x},0)=-\nabla\times\hat{\mathbf{E}}(\mathbf{x},z). \tag{19}$$

Expressing the $\hat{\mathbf{H}}(\mathbf{x},z)$ value from equation (19) and substituting it into equation (18), we obtain:

$$\mathbf{L}^e(\mathbf{x},z)\cdot\hat{\mathbf{E}}(\mathbf{x},z)=\mathbf{g}^e(\mathbf{x},z), \tag{20}$$

where

$$\mathbf{L}^e(\mathbf{x},z)\cdot\hat{\mathbf{E}}(\mathbf{x},z)=z^2\boldsymbol{\varepsilon}(\mathbf{x},z)\hat{\mathbf{E}}(\mathbf{x},z)-\nabla\times\left[\boldsymbol{\mu}^{-1}(\mathbf{x},z)\cdot\nabla\times\hat{\mathbf{E}}(\mathbf{x},z)\right], \tag{21}$$

$$\mathbf{g}^e(\mathbf{x},z)=iz\mathbf{E}(\mathbf{x},0)-\nabla\times\left[\boldsymbol{\mu}^{-1}(\mathbf{x},z)\cdot\mathbf{H}(\mathbf{x},0)\right], \tag{22}$$

$\mathbf{L}^e(z)$ is the electric Helmholtz operator, $\mathbf{g}^e(\mathbf{x},z)$ is a function of the initial electric field configuration. Let us introduce the $\mathbf{G}^e(\mathbf{x},\mathbf{y},z)$ electric Green's function [8] that satisfies

$$\mathbf{L}^e(\mathbf{x},z)\cdot\mathbf{G}^e(\mathbf{x},\mathbf{y},z)=\delta(\mathbf{x}-\mathbf{y})\mathbf{U}, \tag{23}$$

where $\delta(\mathbf{x}-\mathbf{y})$ is the Dirac delta function, $\mathbf{U}$ is the 3×3 unit matrix. Then, the $\mathbf{E}(\mathbf{x},t)$ electric field function is given by the inverse Laplace transform (8) of

$$\hat{\mathbf{E}}(\mathbf{x},z)=\int\mathbf{G}^e(\mathbf{x},\mathbf{y},z)\cdot\mathbf{g}^e(\mathbf{y},z)d\mathbf{y}. \tag{24}$$

Note, that the magnetic Green's function $\mathbf{G}^m(\mathbf{x},\mathbf{y},z)$ and the magnetic field function $\mathbf{H}(\mathbf{x},t)$ can be obtained in a similar way [8]. Therefore, below we consider only electric Green's function. We drop the *e* superscript for brevity, and use sometimes the "Green's function" notion without the word "electric".

2.b. Electric Green's function for the layered system

We study the NIM system composed of (n+m+1) parallel layers, where $n, m \geq 3$ are natural odd integers (Fig. 1). $\mathbf{e}_1$, $\mathbf{e}_2$ unit vectors set a plane of layer's surfaces. An x axis is located parallel to the normal of layer's surfaces, and collinear to the $\mathbf{e}_3$ unit vector. Let us denote a coordinate of the left surface in a k-th layer as x_k, a coordinate of the left surface in the zero layer as $x_0 = 0$. From the right side of the zero layer, n layers are located, from the left side there are m layers. Thus we have $k=-m,\ldots 0,\ldots n$. Layers alternate with each other. All even layers (as well as the zero layer) are Δ_1 in width, and filled with a NIM. All odd layers are Δ_2 in width, and filled with a vacuum. The last-to-left-side (with $-m$ index) and last-to-right-side (with n index) layers are the half spaces unbounded along the direction of the x axis ($x_{-m}=-\infty$, $x_{n+1}=+\infty$) and are empty (vacuum). The point source is located at y coordinate in the zero layer, i.e. $x_0<y<x_1$ or $0<y<\Delta_1$. We assume a translation invariance along the plane of layer's surfaces.

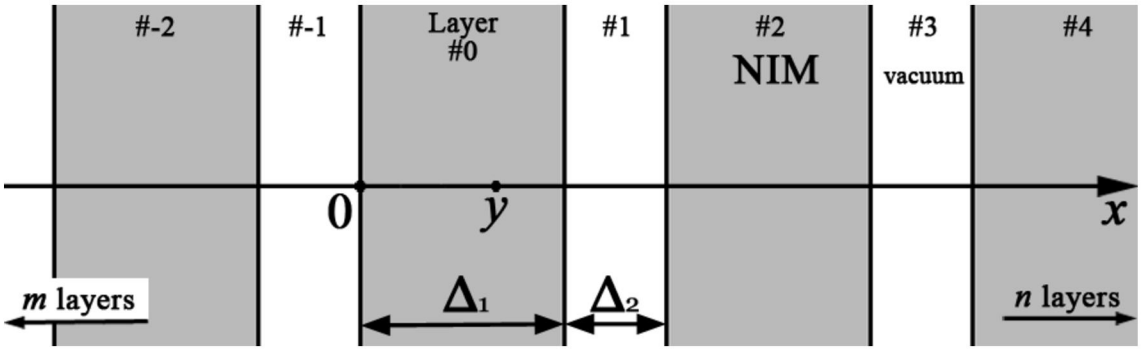

Fig. 1. The NIM system composed of (n+m+1) parallel layers filled with NIM and vacuum. The point source is located at y coordinate in zero NIM layer.

We consider NIM in layers isotropic homogeneous. Therefore, the electric permittivity and magnetic permeability do not depend on coordinates of the $\mathbf{x}$ vector, i.e. $\boldsymbol{\varepsilon}(\mathbf{x},z)=\varepsilon(z)\mathbf{U}$, $\boldsymbol{\mu}(\mathbf{x},z)=\mu(z)\mathbf{U}$. Also, we assume that NIM is a dispersive, non-absorptive medium. In that case, the susceptibilities consist of a sum of Lorentz contributions [16]. We deal with a single dispersive Lorentz contribution [8]. The electric permittivity and magnetic permeability are

$$\varepsilon(z)=\mu(z)=1-\frac{\Omega^2}{z^2-\omega_0^2}, \tag{25}$$

where Ω, ω_0 are constants, $z=\omega+i\alpha$, $\alpha\to 0$, $\alpha>0$, and $\varepsilon(\pm\hat{\omega})=\mu(\pm\hat{\omega})=-1$ for the $\hat{\omega}=\sqrt{\omega_0^2+\Omega^2/2}$ NIM frequency in layers filled with NIM, and $\varepsilon(z)=\mu(z)=1$ in layers filled with a vacuum.

Let $\boldsymbol{\kappa}=\kappa\mathbf{e}_{\boldsymbol{\kappa}}$ be a two-dimensional wave vector, κ be a coordinate of the $\boldsymbol{\kappa}$ vector, $\mathbf{e}_{\boldsymbol{\kappa}}$ be a unit vector parallel to the plane of layer's surfaces. Therefore, $\mathbf{e}_3\times\mathbf{e}_{\boldsymbol{\kappa}}$ is a unit vector parallel to the plane of layer's surfaces and a set of $\mathbf{e}_{\boldsymbol{\kappa}}$, $\mathbf{e}_3\times\mathbf{e}_{\boldsymbol{\kappa}}$, $\mathbf{e}_3$ unit vectors forms the Cartesian basis. Thus, the $\mathbf{e}'=\begin{pmatrix}\mathbf{e}_{\boldsymbol{\kappa}} & \mathbf{e}_3\times\mathbf{e}_{\boldsymbol{\kappa}} & \mathbf{e}_3\end{pmatrix}$ basis is given from the $\mathbf{e}=\begin{pmatrix}\mathbf{e}_1 & \mathbf{e}_2 & \mathbf{e}_3\end{pmatrix}$ basis with a rotation transform by $\gamma_{\boldsymbol{\kappa}}$ angle about the x axis:

$$\mathbf{e}'=\mathbf{e}\cdot\mathbf{T}_{\boldsymbol{\kappa}}, \tag{26}$$

$$\mathbf{T}_{\boldsymbol{\kappa}}=\begin{pmatrix}\cos\gamma_{\boldsymbol{\kappa}} & -\sin\gamma_{\boldsymbol{\kappa}} & 0\\ \sin\gamma_{\boldsymbol{\kappa}} & \cos\gamma_{\boldsymbol{\kappa}} & 0\\ 0 & 0 & 1\end{pmatrix},\ \mathbf{T}_{\boldsymbol{\kappa}}^{-1}=\begin{pmatrix}\cos\gamma_{\boldsymbol{\kappa}} & \sin\gamma_{\boldsymbol{\kappa}} & 0\\ -\sin\gamma_{\boldsymbol{\kappa}} & \cos\gamma_{\boldsymbol{\kappa}} & 0\\ 0 & 0 & 1\end{pmatrix}. \tag{27}$$

Let $\perp$ be a superscript for the vector, which is normal to the $\mathbf{e}_3$ unit vector, i.e., $\mathbf{x}^{\perp}$ is the projection of the $\mathbf{x}$ vector to the plane of layer's surfaces.

Let us consider the Fourier transform for the $\mathbf{G}(\mathbf{x},\mathbf{y},z)$ Green's function with the coordinates corresponding with the $\mathbf{e}_1$ and $\mathbf{e}_2$ unit vectors [8]:

$$\mathbf{G}(\mathbf{x},\mathbf{y},z)=(2\pi)^{-2}\int e^{-i\boldsymbol{\kappa}\cdot(\mathbf{x}^{\perp}-\mathbf{y}^{\perp})}\mathbf{G}_{\boldsymbol{\kappa}}(x,y,z)d\boldsymbol{\kappa}, \tag{28}$$

$$\mathbf{G}_{\boldsymbol{\kappa}}(x,y,z)=\int e^{i\boldsymbol{\kappa}\cdot(\mathbf{x}^{\perp}-\mathbf{y}^{\perp})}\mathbf{G}(\mathbf{x},\mathbf{y},z)d\mathbf{x}^{\perp}, \tag{29}$$

$$\mathbf{m}(\mathbf{x})=(2\pi)^{-2}\int e^{-i\boldsymbol{\kappa}\cdot\mathbf{x}^{\perp}}\mathbf{m}_{\boldsymbol{\kappa}}(x)d\boldsymbol{\kappa}, \tag{30}$$

$$\mathbf{m}_{\boldsymbol{\kappa}}(x)=\int e^{i\boldsymbol{\kappa}\cdot\mathbf{x}^{\perp}}\mathbf{m}(\mathbf{x})d\mathbf{x}^{\perp}, \tag{31}$$

where $\mathbf{m}$ can be $\hat{\mathbf{E}}(\mathbf{x},z)$, $\mathbf{E}(\mathbf{x},t)$, $\hat{\mathbf{H}}(\mathbf{x},z)$, etc. Then, the electric field function is expressed in terms of the Green's function as follows:

$$\mathbf{E}(\mathbf{x},t)=(2\pi)^{-1}\int_{\Gamma}e^{-izt}\hat{\mathbf{E}}(\mathbf{x},z)\,dz=$$

$$=(2\pi)^{-1}\int_{\Gamma}e^{-izt}\,dz\int\mathbf{G}(\mathbf{x},\mathbf{y},z)\cdot\mathbf{g}(\mathbf{y},z)\,d\mathbf{y}=$$

$$=(2\pi)^{-3}\int_{\Gamma}e^{-izt}\,dz\int e^{-i\boldsymbol{\kappa}\cdot(\mathbf{x}^{\perp}-\mathbf{y}^{\perp})}\,d\boldsymbol{\kappa}\int\mathbf{G}_{\boldsymbol{\kappa}}(x,y,z)\cdot\mathbf{g}_{\boldsymbol{\kappa}}(y,z)\,dy\,, \tag{32}$$

where (22) is presented as follows:

$$\mathbf{g}_{\boldsymbol{\kappa}}(y,z)=iz\mathbf{E}_{\boldsymbol{\kappa}}(y,0)-(i\boldsymbol{\kappa}+\frac{\partial}{dy}\mathbf{e}_3)\times\frac{1}{\mu(z)}\mathbf{H}_{\boldsymbol{\kappa}}(y,0)\,. \tag{33}$$

Let

$$\zeta^2(z,\kappa)=z^2\varepsilon(z)\mu(z)-\kappa^2\,, \tag{34}$$

then, the Helmholtz equation (20) is presented as follows:

$$\mathbf{L}_{\boldsymbol{\kappa}}(z)\cdot\hat{\mathbf{E}}_{\boldsymbol{\kappa}}(x,z)=\mathbf{g}_{\boldsymbol{\kappa}}(y,z)\,, \tag{35}$$

and the Helmholtz operator (21) in a certain layer has the form:

$$\mathbf{L}_{\boldsymbol{\kappa}}(z)=\frac{1}{\mu(z)}\left(\frac{\partial^2}{\partial x^2}+\zeta^2(z,\kappa)\right)\mathbf{e}_3\times\mathbf{e}_{\boldsymbol{\kappa}}\mathbf{e}_3\times\mathbf{e}_{\boldsymbol{\kappa}}+\frac{1}{\mu(z)}\left(\frac{\partial^2}{\partial x^2}+z^2\varepsilon(z)\mu(z)\right)\mathbf{e}_{\boldsymbol{\kappa}}\mathbf{e}_{\boldsymbol{\kappa}}+\frac{\zeta^2(z,\kappa)}{\mu(z)}\mathbf{e}_3\mathbf{e}_3+$$

$$-\frac{i\kappa}{\mu(z)}\frac{\partial}{\partial x}\left(\mathbf{e}_{\boldsymbol{\kappa}}\mathbf{e}_3+\mathbf{e}_3\mathbf{e}_{\boldsymbol{\kappa}}\right), \tag{36}$$

where $\varepsilon(z)$, $\mu(z)$, $\zeta^2(\kappa,z)$ correspond to the certain layer. We introduce a notation for a componentwise matrix representation. E.g., for a certain $f(x)$ function and $\mathbf{e}_3\mathbf{e}_{\boldsymbol{\kappa}}$ term

$$f(x)\mathbf{e}_3\mathbf{e}_{\boldsymbol{\kappa}}=f(x)\cdot\left[\begin{pmatrix}\mathbf{e}_{\boldsymbol{\kappa}} & \mathbf{e}_3\times\mathbf{e}_{\boldsymbol{\kappa}} & \mathbf{e}_3\end{pmatrix}\cdot\begin{pmatrix}0\\0\\1\end{pmatrix}\right]\cdot\left[\begin{pmatrix}1 & 0 & 0\end{pmatrix}\cdot\begin{pmatrix}\mathbf{e}_{\boldsymbol{\kappa}}\\ \mathbf{e}_3\times\mathbf{e}_{\boldsymbol{\kappa}}\\ \mathbf{e}_3\end{pmatrix}\right]=$$

$$=\begin{pmatrix}\mathbf{e}_{\boldsymbol{\kappa}} & \mathbf{e}_3\times\mathbf{e}_{\boldsymbol{\kappa}} & \mathbf{e}_3\end{pmatrix}\cdot\begin{pmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ f(x) & 0 & 0\end{pmatrix}\cdot\begin{pmatrix}\mathbf{e}_{\boldsymbol{\kappa}}\\ \mathbf{e}_3\times\mathbf{e}_{\boldsymbol{\kappa}}\\ \mathbf{e}_3\end{pmatrix}. \tag{37}$$

Note, that the $\mathbf{e}_3\mathbf{e}_{\boldsymbol{\kappa}}$ term in (37) does not mean the inner product of the $\mathbf{e}_3$ and $\mathbf{e}_{\boldsymbol{\kappa}}$ unit vectors. In terms of bra-ket notation, an $\mathbf{e}_3\mathbf{e}_{\boldsymbol{\kappa}}$ term means $|\mathbf{e}_3\rangle\langle\mathbf{e}_{\boldsymbol{\kappa}}|$. Then, (23) is presented as follows:

$$\mathbf{L}_{\boldsymbol{\kappa}}(z)\cdot\mathbf{G}_{\boldsymbol{\kappa}}(x,y,z)=\delta(x-y)\mathbf{U}\,. \tag{38}$$

Formulae for (33) and (36) are obtained in the $\{\mathbf{e}_{\boldsymbol{\kappa}},\mathbf{e}_3\times\mathbf{e}_{\boldsymbol{\kappa}},\mathbf{e}_3\}$ basis. Therefore, we need to obtain the $\mathbf{G}_{\boldsymbol{\kappa}}(x,y,z)$ Green's function from (38) in the basis, too. The Green's function in the $\{\mathbf{e}_1,\mathbf{e}_2,\mathbf{e}_3\}$ basis can be obtained by the rotation transform (27). The incident plane is formed by $\mathbf{e}_{\boldsymbol{\kappa}}$ and $\mathbf{e}_3$ unit vectors. Thus, all matrices can be expressed in terms of the *s*- and *p*-polarization parts. According to [8]

$$\mathbf{L}_{\boldsymbol{\kappa}}=\mathbf{L}_{\boldsymbol{\kappa}}^{s}+\mathbf{L}_{\boldsymbol{\kappa}}^{p}\,, \tag{39}$$

i.e. $\mathbf{L}_{\boldsymbol{\kappa}}$ is resolved into $\mathbf{L}_{\boldsymbol{\kappa}}^{s}$, that is the *s*-polarization part (the term with $\mathbf{e}_3\times\mathbf{e}_{\boldsymbol{\kappa}}\mathbf{e}_3\times\mathbf{e}_{\boldsymbol{\kappa}}$ in (36)), and $\mathbf{L}_{\boldsymbol{\kappa}}^{p}$, that is the *p*-polarization part (the rest of terms in (36)). Then, the $\mathbf{G}_{\boldsymbol{\kappa}}(x,y,z)$ Green's function in the decomposition (32) is presented as follows [8]:

$$\mathbf{G}_{\boldsymbol{\kappa}}(x,y,z)=\mathbf{G}^{s}(x,y,z,\kappa)+\mathbf{G}^{p}(x,y,z,\kappa), \tag{40}$$

$$\mathbf{G}^{s}(x,y,z,\kappa)=G^{s}(x,y,z,\kappa)\mathbf{e}_3\times\mathbf{e}_{\boldsymbol{\kappa}}\mathbf{e}_3\times\mathbf{e}_{\boldsymbol{\kappa}}, \tag{41}$$

$$\mathbf{G}^{p}(x,y,z,\kappa)=\left(\mathbf{e}_{\boldsymbol{\kappa}}+\frac{i\kappa}{\zeta^2(x,z,\kappa)}\frac{\partial}{\partial x}\mathbf{e}_3\right)\left(\mathbf{e}_{\boldsymbol{\kappa}}-\frac{i\kappa}{\zeta^2(y,z,\kappa)}\frac{\partial}{\partial y}\mathbf{e}_3\right)G^{p}(x,y,z,\kappa), \tag{42}$$

where $\zeta^2(x,\kappa,z)$ is presented in a layer that corresponds with the *x* coordinate, $\zeta^2(y,\kappa,z)$ is presented in the zero layer, and scalar G^s and G^p functions satisfy the following differential equations:

$$\frac{1}{\mu(z)}\left(\frac{\partial^2}{\partial x^2}+\zeta^2(z,\kappa)\right)G^{s}(x,y,z,\kappa)=\delta(x-y), \tag{43}$$

$$\frac{z^2\varepsilon(z)}{\zeta^2(z,\kappa)}\left(\frac{\partial^2}{\partial x^2}+\zeta^2(z,\kappa)\right)G^{p}(x,y,z,\kappa)=\delta(x-y). \tag{44}$$

2.c. Boundary conditions

We study the system composed of layers divided by plane unbounded surfaces. The boundary conditions for each one are presented in the general form as follows:

$$(\mathbf{E}_1-\mathbf{E}_2)\times\mathbf{n}=\mathbf{0}, \tag{45}$$

$$(\mathbf{H}_1-\mathbf{H}_2)\times\mathbf{n}=\mathbf{0}, \tag{46}$$

$$(\mathbf{D}_1-\mathbf{D}_2)\cdot\mathbf{n}=0, \tag{47}$$

$$(\mathbf{B}_1-\mathbf{B}_2)\cdot\mathbf{n}=0. \tag{48}$$

$\mathbf{E}_j=\mathbf{E}_j(\tilde{\mathbf{x}},t)$, $\mathbf{H}_j=\mathbf{H}_j(\tilde{\mathbf{x}},t)$, $\mathbf{D}_j=\mathbf{D}_j(\tilde{\mathbf{x}},t)$, and $\mathbf{B}_j=\mathbf{B}_j(\tilde{\mathbf{x}},t)$ stand for the limits with $\mathbf{x}\to\tilde{\mathbf{x}}$, where $\tilde{\mathbf{x}}$ is a vector located parallel to the plane of layer's surfaces, *j* is an index distinguishing limits calculated on different surface sides ($j=1,2$), $\mathbf{n}$ is an unit vector of normal to the plane of layer's surfaces ($\mathbf{n}=\mathbf{e}_3$). The conditions (45), (46), (47) after Laplace transform (8) being applied are presented as follows:

$$\left(\hat{\mathbf{E}}_1(\tilde{\mathbf{x}},z)-\hat{\mathbf{E}}_2(\tilde{\mathbf{x}},z)\right)\times\mathbf{e}_3=\mathbf{0}, \tag{49}$$

$$\left(\hat{\mathbf{H}}_1(\tilde{\mathbf{x}},z)-\hat{\mathbf{H}}_2(\tilde{\mathbf{x}},z)\right)\times\mathbf{e}_3=\mathbf{0}, \tag{50}$$

$$\left(\varepsilon_1(z)\hat{\mathbf{E}}_1(\tilde{\mathbf{x}},z)-\varepsilon_2(z)\hat{\mathbf{E}}_2(\tilde{\mathbf{x}},z)\right)\cdot\mathbf{e}_3=0. \tag{51}$$

It is proper to represent the boundary conditions for the *s*- and *p*- polarization parts in the $\{\mathbf{e}_{\boldsymbol{\kappa}},\mathbf{e}_3\times\mathbf{e}_{\boldsymbol{\kappa}},\mathbf{e}_3\}$ basis obtained from the $\{\mathbf{e}_1,\mathbf{e}_2,\mathbf{e}_3\}$ basis with the rotation transform (27). Note, that equations (49), (50), (51) are invariant in respect to the rotation transform (27), i.e. an equality of components located parallel to the plane of layer's surfaces or collinear to the $\mathbf{e}_3$ unit vector, does not depend on the $\gamma_{\boldsymbol{\kappa}}$ rotation angle. Thus,

$$\left(\hat{\mathbf{E}}_1(\tilde{\mathbf{x}},z)-\hat{\mathbf{E}}_2(\tilde{\mathbf{x}},z)\right)\Big|_{\mathbf{e}_\kappa}=0, \tag{52}$$

$$\left(\hat{\mathbf{H}}_1(\tilde{\mathbf{x}},z)-\hat{\mathbf{H}}_2(\tilde{\mathbf{x}},z)\right)\Big|_{\mathbf{e}_\kappa}=0, \tag{53}$$

$$\left(\hat{\mathbf{E}}_1(\tilde{\mathbf{x}},z)-\hat{\mathbf{E}}_2(\tilde{\mathbf{x}},z)\right)\Big|_{\mathbf{e}_3\times\mathbf{e}_\kappa}=0, \tag{54}$$

$$\left(\varepsilon_1(z)\hat{\mathbf{E}}_1(\tilde{\mathbf{x}},z)-\varepsilon_2(z)\hat{\mathbf{E}}_2(\tilde{\mathbf{x}},z)\right)\Big|_{\mathbf{e}_3}=0\,, \tag{55}$$

where for a certain $\mathbf{A}$ vector, $\mathbf{A}\big|_{\mathbf{e}}$ notation means its projection on the $\mathbf{e}$ unit vector.

Let us consider conditions (52), (55) for the *p*-polarization case. The $\hat{\mathbf{E}}_{\boldsymbol{\kappa}}(x,z)$ electric field function from equation (35) is the result of the Fourier transform (30) with coordinates of the $\boldsymbol{\kappa}$ vector located parallel to the plane of layer's surfaces, i.e. with the coordinates corresponding to the $\mathbf{e}_1$ and $\mathbf{e}_2$ unit vectors. Thus,

$$\left(\hat{\mathbf{E}}_{1,\kappa}(\tilde{x},z)-\hat{\mathbf{E}}_{2,\kappa}(\tilde{x},z)\right)\Big|_{\mathbf{e}_\kappa}=0\,, \tag{56}$$

$$\left(\varepsilon_1(z)\hat{\mathbf{E}}_{1,\kappa}(\tilde{x},z)-\varepsilon_2(z)\hat{\mathbf{E}}_{2,\kappa}(\tilde{x},z)\right)\Big|_{\mathbf{e}_3}=0\,, \tag{57}$$

where $\tilde{x}$ is the $\tilde{\mathbf{x}}$ vector's coordinate corresponding to the $\mathbf{e}_3$ unit vector. Then, according to the decomposition (32), the boundary conditions for the *p*-polarization part of the Green's function are given as follows:

$$\left[\left(\mathbf{G}_1^p(\tilde{x},y,z,\kappa)-\mathbf{G}_2^p(\tilde{x},y,z,\kappa)\right)\cdot\mathbf{g}_\kappa(y,z)\right]_{\mathbf{e}_\kappa}=0\,, \tag{58}$$

$$\left[\left(\varepsilon_1(z)\mathbf{G}_1^p(\tilde{x},y,z,\kappa)-\varepsilon_2(z)\mathbf{G}_2^p(\tilde{x},y,z,\kappa)\right)\cdot\mathbf{g}_\kappa(y,z)\right]_{\mathbf{e}_3}=0\,. \tag{59}$$

Equations (58), (59) are valid for any function of initial field configuration (33). Therefore, the first row of the $\mathbf{G}_1^p(\tilde{x},y,z,\kappa)$ tensor function corresponding to the $\mathbf{e}_{\boldsymbol{\kappa}}$ unit vector is completely equal to the first row of $\mathbf{G}_2^p(\tilde{x},y,z,\kappa)$, and the third row of $\mathbf{G}_1^p(\tilde{x},y,z,\kappa)$ corresponding to the $\mathbf{e}_3$ unit vector is equal to the third row of $\mathbf{G}_2^p(\tilde{x},y,z,\kappa)$ within the accuracy of the $\varepsilon_2(z)/\varepsilon_1(z)$ term. Thus, we obtain the boundary conditions for the *p*-polarization part of the Green's function:

$$G_1^p(\tilde{x},y,z,\kappa)=G_2^p(\tilde{x},y,z,\kappa)\,, \tag{60}$$

$$\frac{\partial G_1^p}{\partial x}(\tilde{x},y,z,\kappa)=\frac{\varepsilon_2(z)}{\varepsilon_1(z)}\frac{\zeta_1^{\,2}(z,\kappa)}{\zeta_2^{\,2}(z,\kappa)}\frac{\partial G_2^p}{\partial x}(\tilde{x},y,z,\kappa)\,. \tag{61}$$

Now, let us consider the *s*-polarization case. It can be shown that equation (19) is invariant to the rotation transform (27). From (19) we can express

$$\hat{\mathbf{H}}(\mathbf{x},z)=\frac{1}{iz\mu(z)}\left(\nabla_{\mathbf{e}'}\times\hat{\mathbf{E}}(\mathbf{x},z)-\mathbf{H}(\mathbf{x},0)\right), \tag{62}$$

where $\nabla_{\mathbf{e}'}$ is the Helmholtz operator acting on the $\mathbf{e}'=\left(\mathbf{e}_{\boldsymbol{\kappa}}\quad \mathbf{e}_3\times\mathbf{e}_{\boldsymbol{\kappa}}\quad \mathbf{e}_3\right)$ basis. From (62) and (53) we obtain

$$\left(\nabla_{\mathbf{e}'}\times\left[\hat{\mathbf{E}}_1(\mathbf{x},z)-\frac{\mu_1(z)}{\mu_2(z)}\hat{\mathbf{E}}_2(\mathbf{x},z)\right]\right)\Bigg|_{\mathbf{e}_\kappa}^{\mathbf{x}=\tilde{\mathbf{x}}}=\left(\mathbf{H}_1(\tilde{\mathbf{x}},0)-\frac{\mu_1(z)}{\mu_2(z)}\mathbf{H}_2(\tilde{\mathbf{x}},0)\right)\Bigg|_{\mathbf{e}_\kappa}, \tag{63}$$

where $f(\mathbf{x})\big|^{\mathbf{x}=\tilde{\mathbf{x}}}=f(\tilde{\mathbf{x}})$ for a certain $f(\mathbf{x})$ function. For the *s*-polarization case $\hat{\mathbf{E}}(\mathbf{x},z)\big|_{\mathbf{e}_3}=0$. Also, $\mathbf{H}_1(\mathbf{x},0)=\mathbf{H}_2(\mathbf{x},0)=0$ for any $\mathbf{x}$ vector ($\mathbf{x}\neq\mathbf{y}$) because in the initial time t_0 (t_0 = 0) the electromagnetic field is absent at any point of the system except the coordinates of the **y** vector (then, with time, the field propagates in the system from the coordinates of the **y** vector). Thus,

$$\frac{\partial}{\partial x}\left(\hat{\mathbf{E}}_1(\mathbf{x},z)-\frac{\mu_1(z)}{\mu_2(z)}\hat{\mathbf{E}}_2(\mathbf{x},z)\right)\Bigg|_{\mathbf{e}_3\times\mathbf{e}_\kappa}^{\mathbf{x}=\tilde{\mathbf{x}}}=0\,. \tag{64}$$

Using the Fourier transform (30) for the conditions (53), (54), we obtain

$$\left(\hat{\mathbf{E}}_{1,\kappa}(\tilde{x},z)-\hat{\mathbf{E}}_{2,\kappa}(\tilde{x},z)\right)\Big|_{\mathbf{e}_3\times\mathbf{e}_\kappa}=0\,, \tag{65}$$

$$\frac{\partial}{\partial x}\left(\hat{\mathbf{E}}_{1,\kappa}(x,z)-\frac{\mu_1(z)}{\mu_2(z)}\hat{\mathbf{E}}_{2,\kappa}(x,z)\right)\Bigg|_{\mathbf{e}_3\times\mathbf{e}_\kappa}^{x=\tilde{x}}=0\,. \tag{66}$$

According to the decomposition (32), the boundary conditions for the *s*-polarization part of the Green's function are given as follows:

$$\left[\left(\mathbf{G}_1^s(\tilde{x},y,z,\kappa)-\mathbf{G}_2^s(\tilde{x},y,z,\kappa)\right)\cdot\mathbf{g}_\kappa(y,z)\right]\Big|_{\mathbf{e}_3\times\mathbf{e}_\kappa}=0\,, \tag{67}$$

$$\left[\left(\mu_2(z)\frac{\partial\mathbf{G}_1^s}{\partial x}(\tilde{x},y,z,\kappa)-\mu_1(z)\frac{\partial\mathbf{G}_2^s}{\partial x}(\tilde{x},y,z,\kappa)\right)\cdot\mathbf{g}_\kappa(y,z)\right]\Bigg|_{\mathbf{e}_3\times\mathbf{e}_\kappa}=0\,. \tag{68}$$

Equations (67), (68) are correct for any function of the initial field configuration (33). Therefore, the second row of the $\mathbf{G}_1^s(\tilde{x},y,z,\kappa)$ tensor function corresponding to the $\mathbf{e}_3\times\mathbf{e}_\kappa$ unit vector is completely equal to the second row of $\mathbf{G}_2^s(\tilde{x},y,z,\kappa)$, and the second row of $\frac{\partial\mathbf{G}_1^s}{\partial x}(\tilde{x},y,z,\kappa)$ is equal to the second row of $\frac{\partial\mathbf{G}_2^s}{\partial x}(\tilde{x},y,z,\kappa)$ within the accuracy of $\mu_1(z)/\mu_2(z)$ term. Thus, we obtain the boundary conditions for the *s*-polarization part of the Green's function:

$$G_1^s(\tilde{x},y,z,\kappa)=G_2^s(\tilde{x},y,z,\kappa)\,, \tag{69}$$

$$\frac{\partial G_1^s}{\partial x}(\tilde{x},y,z,\kappa)=\frac{\mu_1(z)}{\mu_2(z)}\frac{\partial G_2^s}{\partial x}(\tilde{x},y,z,\kappa)\,. \tag{70}$$

2.d. Scalar electric Green's function with boundary conditions

Let us consider equation (44). Note, that the solutions of the equation in every layer are well-known. Therefore, the Green's function is obtained through the fundamental system of solutions of equation (44) with the G_1 and G_2 coefficients:

$$G^p(x,y,z,\kappa)=G_1e^{i\zeta x}+G_2e^{-i\zeta x}, \tag{71}$$

where $\zeta=\zeta(z,\kappa)$ is given in the corresponding layer. The coefficients $G_1=G_1(y,z,\kappa)$ and $G_2=G_2(y,z,\kappa)$ are the functions of *y*, *z*, and *κ* variables but we write them as G_1 and G_2 for simplicity (this also holds true for another functions of *y*, *z*, and *κ* variables). These coefficients satisfy a system of equations obtained from the standard boundary conditions for all layers. Formally, the solutions of that system are obtained in the general form (by the Cramer's theorem or by another way). However, it is difficult to analyze the solutions in such a form. A much more useful way is the construction of a system of recurrence relations for the coefficients of the Green's function in every layer. Physically, it means the consideration of the serial reflections. The method is evident in usage and easy in analysis of obtained solutions. In our investigation we choose exactly that way.

The general solution (71) of equation (44) is composed of the two $e^{i\zeta x}$ and $e^{-i\zeta x}$ travelling waves with G_1 and G_2 coefficients. We get three travelling waves at the stage of propagation of travelling wave from the medium with conventional index 1 to the medium with conventional index 2 through the layer surface in the positive direction of the *x* axis:

$$G_1^p(x,y,z,\kappa)=G_0e^{i\zeta_1x}+G_1e^{-i\zeta_1x}, \tag{72}$$

$$G_2^p(x,y,z,\kappa) = G_2 e^{i\zeta_2 x}. \tag{73}$$

The boundary conditions (60), (61) on the layer surface in the $\tilde{x}$ coordinate lead to the following relations:

$$E_0 = \frac{\sigma_{1,2}^+}{2} E_2, \tag{74}$$

$$E_1 = \frac{\sigma_{1,2}^-}{2} E_2, \tag{75}$$

$$\sigma_{k,l}^{\pm} = \frac{\varepsilon_k \zeta_l \pm \varepsilon_l \zeta_k}{\varepsilon_k \zeta_l}, \tag{76}$$

where $E_0 = G_0 e^{i\zeta_1 \tilde{x}}$ is the incident wave, $E_1 = G_1 e^{-i\zeta_1 \tilde{x}}$ is the reflective wave, $E_2 = G_2 e^{i\zeta_2 \tilde{x}}$ is the passed wave, $\varepsilon = \varepsilon(z)$ and $\zeta = \zeta(z,\kappa)$ are calculated in the corresponding medium, $\sigma_{kl}^{\pm} = \sigma_{kl}^{\pm}(z,\kappa)$. There are similar relations between the incident, reflective, and passed waves during the reverse wave propagation through the surface from the medium 2 to the medium 1:

$$E_0 = \frac{\sigma_{2,1}^+}{2} E_2, \tag{77}$$

$$E_1 = \frac{\sigma_{2,1}^-}{2} E_2. \tag{78}$$

Thus, the boundary conditions are considered twice on every layer's surface, for the waves propagated in the positive direction of the *x* axis, and the negative. The scalar *p*-polarized part of the Green's function in the *k*-th layer (excluding the zero layer with the point source) is given as follows:

$$G_k^p(x,y,z,\kappa) = \left(A_k + B_k\right) e^{i\zeta_k x} + \left(C_k + D_k\right) e^{-i\zeta_k x}, \tag{79}$$

where A_k and D_k are the coefficients of the waves passed into the *k*-th layer through the left and right surface, correspondingly, B_k and C_k are the coefficients of reflective waves inside the *k*-th layer from the left and right surface correspondingly.

There is a point source in the *y* coordinate in the zero layer. The waves progressing to the left side $I_- e^{-i\zeta_0 x}$ and to the right side $I_+ e^{i\zeta_0 x}$ from the point source, are the solutions of equation (44), too. So, to the left side of the zero layer

$$G_{0,-}^p(x,y,z,\kappa) = \left(A_0 + B_0\right) e^{i\zeta_0 x} + \left(C_0 + D_0 + I_-\right) e^{-i\zeta_0 x}, \tag{80}$$

and to the right side of the zero layer

$$G_{0,+}^p(x,y,z,\kappa) = \left(A_0 + B_0 + I_+\right) e^{i\zeta_0 x} + \left(C_0 + D_0\right) e^{-i\zeta_0 x}. \tag{81}$$

The I_- and I_+ coefficients are given by the continuity condition of the Green's function and the condition of a discontinuous jump of the Green's function's derivative in the *y* coordinate:

$$G_{0,-}^p(y,y,z,\kappa) = G_{0,+}^p(y,y,z,\kappa), \tag{82}$$

$$\frac{\partial G_{0,+}^p}{\partial x}(y,y,z,\kappa) - \frac{\partial G_{0,-}^p}{\partial x}(y,y,z,\kappa) = \frac{z^2 \varepsilon_0(z)}{\zeta_0^2(z,\kappa)}, \tag{83}$$

where $G_{0,-}^p(y,y,z,\kappa)$ and $G_{0,+}^p(y,y,z,\kappa)$ means left-sided and right-sided limits with $x \to y$. Then, in the zero layer

$$G_0^p(x,y,z,\kappa) = \left(A_0 + B_0\right) e^{i\zeta_0 x} + \left(C_0 + D_0\right) e^{-i\zeta_0 x} + I_0 e^{-i\zeta_0 |x-y|}, \tag{84}$$

$$I_0 = \frac{\zeta_0(z,\kappa)}{2iz^2\varepsilon_0(z)}. \tag{85}$$

Physically, the waves progressing only away from the point source can be solutions of equation (44) in the last-to-left and last-to-right side of layers, i.e. in the layers with n and $-m$ indices:

$$G_n^p(x,y,z,\kappa) = A_n e^{i\zeta_n x}, \tag{86}$$

$$G_{-m}^p(x,y,z,\kappa) = D_{-m} e^{-i\zeta_{-m} x}. \tag{87}$$

The travelling waves passing through the surfaces in x_n and $x_{-(m-1)}$ coordinates into the layers adjoining to the last layers, are absent in the solutions (79), i.e. in the layers with (n–1) and –(m–1) indices

$$G_{n-1}^p(x,y,z,\kappa) = (A_{n-1} + B_{n-1}) e^{i\zeta_{n-1} x} + C_{n-1} e^{-i\zeta_{n-1} x}, \tag{88}$$

$$G_{-(m-1)}^p(x,y,z,\kappa) = B_{-(m-1)} e^{i\zeta_{-(m-1)} x} + (C_{-(m-1)} + D_{-(m-1)}) e^{-i\zeta_{-(m-1)} x}. \tag{89}$$

As a result, the scalar p-polarized part of the Green's function is expressed as follows:

$$G^p(x,y,z,\kappa) = \begin{cases} D_{-m} e^{-i\zeta_{-m} x} & x \in (-\infty, x_{-(m-1)}) \\ B_{-(m-1)} e^{i\zeta_{-(m-1)} x} + C_{-(m-1)} e^{-i\zeta_{-(m-1)} x} + D_{-(m-1)} e^{-i\zeta_{-(m-1)} x} & x \in (x_{-(m-1)}, x_{-(m-2)}) \\ A_{-(m-2)} e^{i\zeta_{-(m-2)} x} + B_{-(m-2)} e^{i\zeta_{-(m-2)} x} + C_{-(m-2)} e^{-i\zeta_{-(m-2)} x} + D_{-(m-2)} e^{-i\zeta_{-(m-2)} x} & x \in (x_{-(m-2)}, x_{-(m-3)}) \\ \dots \quad \dots \quad \dots \quad \dots & \dots \\ A_{-1} e^{i\zeta_{-1} x} + B_{-1} e^{i\zeta_{-1} x} + C_{-1} e^{-i\zeta_{-1} x} + D_{-1} e^{-i\zeta_{-1} x} & x \in (x_{-1}, 0) \\ A_0 e^{i\zeta_0 x} + B_0 e^{i\zeta_0 x} + C_0 e^{-i\zeta_0 x} + D_0 e^{-i\zeta_0 x} + I_0 e^{-i\zeta_0 |x-y|} & x \in (0, x_1) \\ A_1 e^{i\zeta_1 x} + B_1 e^{i\zeta_1 x} + C_1 e^{-i\zeta_1 x} + D_1 e^{-i\zeta_1 x} & x \in (x_1, x_2) \\ \dots \quad \dots \quad \dots \quad \dots & \dots \\ A_{n-2} e^{i\zeta_{n-2} x} + B_{n-2} e^{i\zeta_{n-2} x} + C_{n-2} e^{-i\zeta_{n-2} x} + D_{n-2} e^{-i\zeta_{n-2} x} & x \in (x_{n-2}, x_{n-1}) \\ A_{n-1} e^{i\zeta_{n-1} x} + B_{n-1} e^{i\zeta_{n-1} x} + C_{n-1} e^{-i\zeta_{n-1} x} & x \in (x_{n-1}, x_n) \\ A_n e^{i\zeta_n x} & x \in (x_n, +\infty) \end{cases}, \tag{90}$$

where $\zeta_k = \zeta_k(z,\kappa)$ (34) is obtained in the k-th layer with $k = -m,\dots,n$, and I_0 is given in (85).

The reasoning similar to the above mentioned one leads to an expression for the scalar s-polarized part of the $G^s(x,y,z,\kappa)$ Green's function, which is the same as in (90). But there are differences. The first one consists in relations (74), (78) for the E_0 incident wave, the E_1 reflective wave, and the E_2 passed wave:

$$E_0 = \frac{\tau_{1,2}^+}{2} E_2, \tag{91}$$

$$E_1 = \frac{\tau_{1,2}^-}{2} E_2, \tag{92}$$

$$\tau_{k,l}^{\pm} = \frac{\mu_l \zeta_k \pm \mu_k \zeta_l}{\mu_l \zeta_k}, \tag{93}$$

where $\mu = \mu(z)$ and $\zeta = \zeta(z,\kappa)$ are obtained in the corresponding medium, $\tau_{kl}^{\pm} = \tau_{kl}^{\pm}(z,\kappa)$. Equation (25) implies

$$\tau_{kl}^{\pm} = \sigma_{lk}^{\pm}. \tag{94}$$

Thus, for the *s*-polarization case, relations (74), (75) and also (77), (78) are valid with changing subscripts for $\sigma_{kl}^{\pm}$.

The last term in expression (85) for the *s*-polarization case has the form

$$I_0 = \frac{\mu_0(z)}{2i\zeta_0(z,\kappa)}. \tag{95}$$

3. Recurrence relations

3.a. Green's function coefficients

Let us consider the scalar *p*-polarized part of the Green's function in the *k*-th layer (79). It is related with the Green's functions in the (*k*–1)-th and (*k*+1)-th layers correspondingly by the boundary conditions (74) and (78) on the left surface and (75) and (77) on the right surface. By considering these conditions for each layer of the system, we obtain expressions for the coefficients of the scalar *p*-polarized part of the Green's function [11]. Thus, for $k = 1,\ldots(n-1)$

$$A_k = \beta_k A_n,\ B_k = \frac{d_k}{c_k}\gamma_k A_n,\ C_k = \frac{h_k}{g_k}\beta_{k+1}A_n,\ D_k = \gamma_{k+1}A_n,\ C_{n-1} = \frac{h_{n-1}}{g_{n-1}}A_n,\ D_{n-1} = 0, \tag{96}$$

for $k = -(m-1),\ldots 0$

$$A_k = \beta_k A_n + \xi_k,\ B_k = \frac{d_k}{c_k}(\gamma_k A_n + \eta_k),\ C_k = \frac{h_k}{g_k}(\beta_{k+1}A_n + \xi_{k+1}),\ D_k = \gamma_{k+1}A_n + \eta_{k+1},$$

$$C_0 = \frac{h_0}{g_0}\beta_1 A_n,\ D_0 = \gamma_1 A_n,\ A_{-(m-1)} = 0,\ D_{-m} = \gamma_{-(m-1)}A_n + \eta_{-(m-1)}. \tag{97}$$

The β_k, γ_k, ξ_k, and η_k values in (96) and (97) satisfy the following recurrence relations [11]:

$$\beta_k = J_k\beta_{k+1} - \left(\frac{a\,d}{b\,c}\right)_k \gamma_{k+1},\ \beta_{n-1} = J_{n-1},\ J_k = \frac{f_k}{e_k}\left(1 - \left(\frac{a\,d\,e\,h}{b\,c\,f\,g}\right)_k\right), \tag{98}$$

$$\gamma_k = \frac{a_k}{b_k}\left(\frac{h_k}{g_k}\beta_{k+1} + \gamma_{k+1}\right),\ \gamma_{n-1} = \left(\frac{a\,h}{b\,g}\right)_{n-1}, \tag{99}$$

$$\xi_k = J_k\xi_{k+1} - \left(\frac{a\,d}{b\,c}\right)_k \eta_{k+1},\ \xi_0 = I_0\left(-e^{-i\zeta_0 y} - \left(\frac{a\,d}{b\,c}\right)_0 e^{i\zeta_0 y}\right), \tag{100}$$

$$\eta_k = \frac{a_k}{b_k}\left(\frac{h_k}{g_k}\xi_{k+1} + \eta_{k+1}\right),\ \eta_0 = I_0\frac{a_0}{b_0}e^{i\zeta_0 y}, \tag{101}$$

where

$$a_k = 2e^{-i\zeta_k x_k},\ b_k = \sigma_{k,k-1}^{+}e^{-i\zeta_{k-1}x_k},\ c_k = 2e^{i\zeta_k x_k},\ d_k = \sigma_{k,k-1}^{-}e^{-i\zeta_{k-1}x_k},$$

$$e_k = 2e^{i\zeta_k x_{k+1}},\ f_k = \sigma_{k,k+1}^{+}e^{i\zeta_{k+1}x_{k+1}},\ g_k = 2e^{-i\zeta_k x_{k+1}},\ h_k = \sigma_{k,k+1}^{-}e^{i\zeta_{k+1}x_{k+1}}, \tag{102}$$

and $\sigma_{kl}^{\pm}$ is defined in (76), I_0 is defined in (85). We introduce the $\left(\frac{a\,d}{b\,c}\right)_k = \frac{a_k d_k}{b_k c_k}$ notation. The relations for β_{n-1} in (98), γ_{n-1} in (99), ξ_0 in (100), and η_0 in (101) are called the initial conditions for the recurrence relations (98)-(101).

All the coefficients (96) and (97) are functions of A_n, which satisfies the following relation [11]:

$$A_n = -\frac{\xi_{-(m-1)}}{\beta_{-(m-1)}}. \tag{103}$$

Thus, the problem of obtaining the scalar *p*-polarized part of the Green's function for the considered NIM system is reduced to the construction of the solution of the recurrence relations (98)-(101) and obtaining the A_n coefficient.

3.b. General solutions of recurrence relations

Note, that relations (98) and (100) have similar structure with different initial conditions. The same takes place for relations (99) and (101). Thus, it is enough to obtain general solutions for relations (98)-(99), and to use then the corresponding initial conditions. The relation for β_k is obtained from (98)-(99) and has the form:

$$\beta_k = K_k \beta_{k+2} - L_k \beta_{k+4}, \tag{104}$$

with the initial conditions

$$\beta_{n-4} = P_{n-4}\beta_{n-2} - Q_{n-4}\gamma_{n-2},\ \beta_{n-3} = P_{n-3}\beta_{n-1} - Q_{n-3}\gamma_{n-1},$$

$$\beta_{n-2} = J_{n-2}\beta_{n-1} - \left(\frac{a\,d}{b\,c}\right)_{n-2}\gamma_{n-1},\ \beta_{n-1} = J_{n-1}. \tag{105}$$

Correspondingly, for γ_k we have

$$\gamma_k = M_k \gamma_{k+2} - N_k \gamma_{k+4}, \tag{106}$$

with the initial conditions

$$\gamma_{n-4} = R_{n-4}\beta_{n-2} + S_{n-4}\gamma_{n-2},\ \gamma_{n-3} = R_{n-3}\beta_{n-1} + S_{n-3}\gamma_{n-1},$$

$$\gamma_{n-2} = \frac{a_{n-2}}{b_{n-2}}\left(\frac{h_{n-2}}{g_{n-2}}\beta_{n-1} + \gamma_{n-1}\right),\ \gamma_{n-1} = \left(\frac{a\,h}{b\,g}\right)_{n-1}. \tag{107}$$

We use the following notations:

$$P_k = J_k J_{k+1} - \left(\frac{a\,d}{b\,c}\right)_k \left(\frac{a\,h}{b\,g}\right)_{k+1},\ Q_k = J_k \left(\frac{a\,d}{b\,c}\right)_{k+1} + \left(\frac{a\,d}{b\,c}\right)_k \left(\frac{a}{b}\right)_{k+1},$$

$$R_k = \left(\frac{a\,h}{b\,g}\right)_k J_{k+1} + \left(\frac{a}{b}\right)_k \left(\frac{a\,h}{b\,g}\right)_{k+1},\ S_k = \left(\frac{a}{b}\right)_k \left(\frac{a}{b}\right)_{k+1} - \left(\frac{a\,h}{bg}\right)_k \left(\frac{a\,d}{b\,c}\right)_{k+1}, \tag{108}$$

$$K_k = P_k + \frac{Q_k S_{k+2}}{Q_{k+2}},\ L_k = Q_k R_{k+2} + \frac{P_{k+2} Q_k S_{k+2}}{Q_{k+2}},$$

$$M_k = S_k + \frac{R_k P_{k+2}}{R_{k+2}},\ N_k = R_k Q_{k+2} + \frac{P_{k+2} R_k S_{k+2}}{R_{k+2}}. \tag{109}$$

Note, that all the layers with the same parity are identical. Therefore, $\zeta_k = \zeta_{k+2}$ for $-m \le k \le (n-2)$, $\sigma^{\pm}_{k+1,l+1} = \sigma^{\pm}_{k-1,l-1}$ and $\sigma^{\pm}_{k,k+1} = \sigma^{\pm}_{k,k-1}$ for $-(m-1) \le k,l \le (n-1)$. For the Δ_k width of the *k*-th layer, we have $\Delta_k = \Delta_{k+2}$ for $-m \le k \le (n-2)$. For the considered system let $\Delta_k = \Delta_1$ for even *k* (and *k* = 0), and $\Delta_k = \Delta_2$ for odd *k*. Then, the expressions (108) are as follows:

$$P_k = \left[\frac{\sigma^{+}_{k,k+1}\sigma^{+}_{k+1,k}}{4} + \frac{\sigma^{-}_{k,k+1}\sigma^{-}_{k+1,k}}{4}\left(e^{i\zeta_k 2\Delta_k} + e^{i\zeta_{k+1} 2\Delta_{k+1}} - e^{i\zeta_k 2\Delta_k + i\zeta_{k+1} 2\Delta_{k+1}}\right)\right] e^{i(\zeta_k - \zeta_{k+1})\Delta_{k+1}}, \tag{110}$$

$$Q_k = -\frac{\sigma^{-}_{k,k+1}}{2}\left(1 - e^{i\zeta_k 2\Delta_k}\right) e^{-i(\zeta_k + \zeta_{k+1})x_{k+1}}, \tag{111}$$

$$R_k = -\frac{\sigma^-_{k+1,k}}{2}\left(1-e^{i\zeta_{k+1}2\Delta_{k+1}}\right)e^{i\zeta_k(x_k+2\Delta_k+\Delta_{k+1})+i\zeta_{k+1}(x_k-\Delta_{k+1})}, \tag{112}$$

$$S_k = e^{i(\zeta_k-\zeta_{k+1})\Delta_k}, \tag{113}$$

where

$$P_{k-2} = P_k,\ S_{k-2} = S_k,\ Q_{k-2} = Q_k e^{i(\zeta_k+\zeta_{k+1})(\Delta_k+\Delta_{k+1})},\ R_{k-2} = R_k e^{-i(\zeta_k+\zeta_{k+1})(\Delta_k+\Delta_{k+1})}. \tag{114}$$

The expressions (109) are as follows:

$$K_k = \left[\frac{\sigma^+_{k,k+1}\sigma^+_{k+1,k}}{4}\left(1+e^{i\zeta_k 2\Delta_k+i\zeta_{k+1}2\Delta_{k+1}}\right)+\frac{\sigma^-_{k,k+1}\sigma^-_{k+1,k}}{4}\left(e^{i\zeta_k 2\Delta_k}+e^{i\zeta_{k+1}2\Delta_{k+1}}\right)\right]e^{i(\zeta_k-\zeta_{k+1})\Delta_{k+1}}, \tag{115}$$

$$L_k = e^{i\zeta_k 2(\Delta_k+\Delta_{k+1})}, \tag{116}$$

$$M_k = \left[\frac{\sigma^+_{k,k+1}\sigma^+_{k+1,k}}{4}\left(1+e^{i\zeta_k 2\Delta_k+i\zeta_{k+1}2\Delta_{k+1}}\right)+\frac{\sigma^-_{k,k+1}\sigma^-_{k+1,k}}{4}\left(e^{i\zeta_k 2\Delta_k}+e^{i\zeta_{k+1}2\Delta_{k+1}}\right)\right]e^{-i\zeta_k\Delta_k-i\zeta_{k+1}(\Delta_k+2\Delta_{k+1})}, \tag{117}$$

$$N_k = e^{-i\zeta_{k+1}2(\Delta_k+\Delta_{k+1})}. \tag{118}$$

We obtain the relations of $K_k = K_{k+2}$, $L_k = L_{k+2}$, $M_k = M_{k+2}$, and $N_k = N_{k+2}$ for $-m \le k \le (n-2)$. Thus, for each K_k, L_k, M_k, N_k, ζ_k, ε_k, and μ_k value there are only two values possessed according to parity of the *k* index. The same is correct for the $\sigma^{\pm}_{k,k+1}$ values. From now on, we use these values with the subscript "$_1$" (e.g., K_1, $\sigma^-_{1,2}$) if *k* is even and with the subscript "$_2$" (e.g., K_2, $\sigma^-_{2,1}$) if *k* is odd. Therefore, equations (104) and (106) for even *k* (and *k* = 0) or for odd *k* are separately considered as the recurrence relations with constant coefficients. The solutions of these relations are obtained by the generating function method.

Let us consider equation (104) with the boundary conditions (105) for even *k* with the following substitution:

$$u_j = \beta_k, \tag{119}$$

where $j = (n-1-k)/2$ and *k* is even.

Thus, for $k = (n-1), (n-3), \ldots,\ j = 0, 1, \ldots,\ u_0 = \beta_{n-1},\ u_1 = \beta_{n-3}$ etc equation (104) becomes

$$u_j = K_1 u_{j-1} - L_1 u_{j-2}, \tag{120}$$

where

$$K_1 = \left[\frac{\sigma^+_{1,2}\sigma^+_{2,1}}{4}\left(1+e^{i\zeta_1 2\Delta_1+i\zeta_2 2\Delta_2}\right)+\frac{\sigma^-_{1,2}\sigma^-_{2,1}}{4}\left(e^{i\zeta_1 2\Delta_1}+e^{i\zeta_2 2\Delta_2}\right)\right]e^{i(\zeta_1-\zeta_2)\Delta_2}, \tag{121}$$

$$L_1 = e^{i\zeta_1 2(\Delta_1+\Delta_2)}, \tag{122}$$

$$u_0 = \frac{\sigma^+_{1,2}}{2}\left(1+\frac{\sigma^-_{1,2}\sigma^-_{2,1}}{\sigma^+_{1,2}\sigma^+_{2,1}}e^{i\zeta_1 2\Delta_1}\right)e^{-i(\zeta_1-\zeta_2)x_n}, \tag{123}$$

$$u_1 = \frac{\sigma^+_{1,2}}{2}\left\{\left[1-\frac{\sigma^-_{1,2}\sigma^-_{2,1}}{4}\left(1-e^{i\zeta_1 2\Delta_1}\right)^2\left(1-e^{i\zeta_2 2\Delta_2}\right)\right]+\frac{\sigma^-_{1,2}\sigma^-_{2,1}}{\sigma^+_{1,2}\sigma^+_{2,1}}e^{i\zeta_1 4\Delta_1}\right\}e^{-i\zeta_1(x_{n-1}+\Delta_1-\Delta_2)+i\zeta_2(x_{n-1}+\Delta_1-\Delta_2)}. \tag{124}$$

The generating function

$$U(t) = \sum_{j=0}^{\infty} u_j t^j \tag{125}$$

for equation (120) is defined as follows:

$$U(t)=\frac{u_0-(K_1u_0-u_1)t}{L_1t^2-K_1t+1}. \tag{126}$$

The rational function from (126) is expanded into the partial fractions

$$U(t)=\frac{U_1}{t-t_1}-\frac{U_2}{t-t_2}, \tag{127}$$

where

$$t_1=\frac{K_1+\left(K_1^2-4L_1\right)^{1/2}}{2L_1},\ t_2=\frac{K_1-\left(K_1^2-4L_1\right)^{1/2}}{2L_1}, \tag{128}$$

$$U_l=\frac{1}{L_1}\left(\frac{u_0-(K_1u_0-u_1)t_l}{t_1-t_2}\right),\ l=1,2. \tag{129}$$

The expansion of each partial fraction (127) into a series is as follows:

$$\frac{1}{t-t_l}=-\frac{1}{t_l}\sum_{j=0}^{\infty}\left(\frac{t}{t_l}\right)^j,\ l=1,2. \tag{130}$$

Comparing the u_j unknown in (125) to the coefficients of corresponding t^j in (126), we obtain the general formula for the solutions of equation (120)

$$u_j=-\frac{1}{2^{j+2}}\frac{1}{L_1\left(K_1^2-4L_1\right)^{1/2}}\left[\begin{array}{l}\left(2L_1u_0-(K_1u_0-u_1)\left(K_1+\left(K_1^2-4L_1\right)^{1/2}\right)\right)\left(K_1-\left(K_1^2-4L_1\right)^{1/2}\right)^{j+1}\\-\left(2L_1u_0-(K_1u_0-u_1)\left(K_1-\left(K_1^2-4L_1\right)^{1/2}\right)\right)\left(K_1+\left(K_1^2-4L_1\right)^{1/2}\right)^{j+1}\end{array}\right], \tag{131}$$

where $j=0,1,\ldots$, K_1 and L_1 are obtained from (121) and (122) correspondingly, u_0, u_1 are obtained from (119) and (105) correspondingly. The expression (131) contains the term

$$\left(K_1^2-4L_1\right)^{1/2}=\left[\left(\frac{\sigma_{1,2}^{+}\sigma_{2,1}^{+}}{4}\left(1+e^{i\zeta_1 2\Delta_1+i\zeta_2 2\Delta_2}\right)+\frac{\sigma_{1,2}^{-}\sigma_{2,1}^{-}}{4}\left(e^{i\zeta_1 2\Delta_1}+e^{i\zeta_2 2\Delta_2}\right)\right)^2-4e^{i\zeta_1 2\Delta_1+i\zeta_2 2\Delta_2}\right]^{1/2}e^{i(\zeta_1-\zeta_2)\Delta_2}. \tag{132}$$

The general formula for the solutions of equation (104) with the boundary conditions (105) for odd k is obtained by using of the following substitution:

$$v_j=\beta_k, \tag{133}$$

where $j=(n-2-k)/2$, k is odd. Equation (106) with the boundary conditions (107) is solved in the similar way as equation (104) with the boundary conditions (105).

Equations (98) and (100), and also (99) and (101), have the identical structure but different initial conditions. Therefore, the solutions of equations (100) and (101) are obtained using the corresponding initial conditions in the way similar to the introduced one. Thus, the solutions of equations (98)-(101) are obtained in the general form by using relations (131), (119), and similar to them.

3.c. NIM situation

We are interested in the NIM situation. For fixed κ, Ω and ω_0, we denote

$$\rho(\omega)=\left|\omega^2\left(1-\frac{\Omega^2}{\omega^2-\omega_0^2}\right)^2-\kappa^2\right|^{1/2}. \tag{134}$$

According to the results of [8], there are two cases for the $\zeta_k(z)=\zeta_k(z,\kappa)$ value (34) obtained in the *k*-th layer with $z\to\pm\hat{\omega}$:

1. *The $\hat{\omega}>\kappa$ case is called "radiative regime". We have $\zeta_k(\pm\hat{\omega})=\mp\rho(\hat{\omega})$ if k is even (or k = 0), $\zeta_k(\pm\hat{\omega})=\pm\rho(\hat{\omega})$ if k is odd, and $\sigma_{1,2}^{-}=\sigma_{1,2}^{-}(z,\kappa)\to 0$ with $z\to\pm\hat{\omega}$.*

2. *The $\hat{\omega}<\kappa$ case is called "evanescent regime". We have $\zeta_k(\pm\hat{\omega})=i\rho(\hat{\omega})$ for any index k, and $\sigma_{1,2}^{+}=\sigma_{1,2}^{+}(z,\kappa)\to 0$ with $z\to\pm\hat{\omega}$.*

We introduce the conventional notation $O\left(\sigma_{1,2}^{\pm}\right)$, which implies a certain value that has the same infinitesimal order as $\sigma_{1,2}^{\pm}$. We noted above (see subsection 3.2) that $\sigma_{k,k+1}^{\pm}$ is equal to $\sigma_{1,2}^{\pm}$ or $\sigma_{2,1}^{\pm}$ according to parity of the *k* index. The $\sigma_{1,2}^{\pm}=\sigma_{1,2}^{\pm}(z,\kappa)$ and $\sigma_{2,1}^{\pm}=\sigma_{2,1}^{\pm}(z,\kappa)$ values tend to zero simultaneously with $z\to\pm\hat{\omega}$ and have the same infinitesimal order, i.e. $\sigma_{2,1}^{\pm}=O\left(\sigma_{1,2}^{\pm}\right)$. Thus, the general solutions of equations (104) and (106) in the NIM situation are expressed in terms of the asymptotic approximations with $\sigma_{1,2}^{\pm}\to 0$.

Let us consider the solutions (131) and (119) of relations (104) with the initial conditions (105) in the NIM situation. With $z\to\pm\hat{\omega}$, the expression (132) simplifies

$$\left(K_1^2-4L_1\right)^{1/2}=e^{i(\zeta_1-\zeta_2)\Delta_2}\left(1-e^{i\zeta_1 2\Delta_1+i\zeta_2 2\Delta_2}\right)\text{ for }\hat{\omega}>\kappa,\ \sigma_{1,2}^{-}\to 0\text{ with }z\to\pm\hat{\omega},\tag{135}$$

$$\left(K_1^2-4L_1\right)^{1/2}=e^{i(\zeta_1-\zeta_2)\Delta_2}\left(e^{i\zeta_1 2\Delta_1}-e^{i\zeta_2 2\Delta_2}\right)\text{ for }\hat{\omega}<\kappa,\ \sigma_{1,2}^{+}\to 0\text{ with }z\to\pm\hat{\omega}.\tag{136}$$

For the $\hat{\omega}<\kappa$ case with $z\to\pm\hat{\omega}$, the expression (131) contains the first order pole:

$$W=\frac{1}{\sigma_{1,2}^{+}}\overset{z\to\pm\hat{\omega}}{\sim}\left(\frac{\rho(\hat{\omega})\Omega}{2\kappa}\right)^2\frac{1}{(z-\hat{\omega})(z+\hat{\omega})}.\tag{137}$$

Therefore, we obtain the expression (131) as asymptotic approximation with $\sigma_{1,2}^{+}\to 0$ ($z\to\pm\hat{\omega}$). From the relation for σ_{kl}^{+} (76) we have the following expressions, which are required for obtaining the below presented results:

$$\sigma_{1,2}^{-}=2-\sigma_{1,2}^{+},\tag{138}$$

$$\sigma_{2,1}^{+}=-\sigma_{1,2}^{+}+O\left(\sigma_{1,2}^{+}\right)^2,\tag{139}$$

$$\sigma_{2,1}^{-}=2+\sigma_{1,2}^{+}+O\left(\sigma_{1,2}^{+}\right)^2,\tag{140}$$

$$\frac{\sigma_{1,2}^{+}\sigma_{2,1}^{+}}{4}=-\frac{\left(\sigma_{1,2}^{+}\right)^2}{4}+O\left(\sigma_{1,2}^{+}\right)^3.\tag{141}$$

The asymptotic approximation for the expression (132) is obtained by the expansion in the Taylor's series in terms of powers of $\sigma_{1,2}^{+}$, where $\sigma_{1,2}^{+}\to 0$ with $z\to\pm\hat{\omega}$.

$$\left(K_1^2-4L_1\right)^{1/2}=e^{i(\zeta_1-\zeta_2)\Delta_2}\left(e^{i\zeta_1 2\Delta_1}-e^{i\zeta_2 2\Delta_2}\right)+O\left(\sigma_{1,2}^{+}\right)^2.\tag{142}$$

The same is correct for other values that are contained in the expression (132):

$$K_1+\left(K_1^2-4L_1\right)^{1/2}=2e^{i\zeta_1 2\Delta_1}e^{i(\zeta_1-\zeta_2)\Delta_2}+O\left(\sigma_{1,2}^{+}\right)^2,\tag{143}$$

$$K_1-\left(K_1^2-4L_1\right)^{1/2}=2e^{i\zeta_2 2\Delta_2}e^{i(\zeta_1-\zeta_2)\Delta_2}+O\left(\sigma_{1,2}^{+}\right)^2,\tag{144}$$

$$u_0 = -2We^{i\zeta_1 2\Delta_1} e^{-i(\zeta_1-\zeta_2)x_n} + O\left(\sigma_{1,2}^{+}\right), \tag{145}$$

$$u_1 = -2We^{-i(\zeta_1-\zeta_2)(x_n-\Delta_2)} e^{i\zeta_1 4\Delta_1} + O\left(\sigma_{1,2}^{+}\right), \tag{146}$$

$$2L_1 u_0 = -4We^{i\zeta_1 2(2\Delta_1+\Delta_2)} e^{-i(\zeta_1-\zeta_2)x_n} + O\left(\sigma_{1,2}^{+}\right), \tag{147}$$

$$K_1 u_0 - u_1 = -2We^{i\zeta_1 2\Delta_1 + i\zeta_2 2\Delta_2} e^{i(\zeta_1-\zeta_2)\Delta_2} e^{-i(\zeta_1-\zeta_2)x_n} + O\left(\sigma_{1,2}^{+}\right). \tag{148}$$

The general solution (131) of the recurrence relation (120) in the NIM situation for $\hat{\omega} < \kappa$ is presented as follows:

$$u_j = -2We^{-i(\zeta_1-\zeta_2)(x_n - j\Delta_2)} e^{i\zeta_1 (j+1)2\Delta_1} + O\left(\sigma_{1,2}^{+}\right) \text{ with } j = 0,1,\ldots. \tag{149}$$

This expression contains the first order pole (137) and grows to infinity with $z \to \pm\hat{\omega}$.

3.d. Recurrence relation solutions in the NIM situation

Using the presented above way (see subsection 3.2, 3.3), we obtain asymptotic approximations in the NIM situation for the β_k, γ_k, ξ_k, and η_k unknowns that satisfy the recurrence relations (98)-(101). For the $\hat{\omega} > \kappa$ case, they are singularity-free, and for even *k* are represented as follows:

$$\beta_k = e^{\pm i\rho((n+1)\Delta_1 + k\Delta_2)} + O\left(\sigma_{1,2}^{-}\right)^2, \; \gamma_k = O\left(\sigma_{1,2}^{-}\right),$$

$$\xi_k = -I_0 e^{\pm i\rho(k\Delta_2 + y)} + O\left(\sigma_{1,2}^{-}\right), \; \eta_k = I_0 e^{\pm i\rho(k\Delta_1 - y)} + O\left(\sigma_{1,2}^{-}\right), \tag{150}$$

for odd *k*:

$$\beta_k = e^{\pm i\rho(n-k)\Delta_1} + O\left(\sigma_{1,2}^{-}\right)^2, \; \gamma_k = O\left(\sigma_{1,2}^{-}\right),$$

$$\xi_k = -I_0 e^{\pm i\rho(-(k+1)\Delta_1 + y)} + O\left(\sigma_{1,2}^{-}\right), \; \eta_k = I_0 e^{\pm i\rho(-(k-1)\Delta_2 - y)} + O\left(\sigma_{1,2}^{-}\right), \tag{151}$$

where $\rho = \rho(\hat{\omega})$ (134), the value (85) is

$$I_0 = \pm \frac{\rho(\hat{\omega})}{2i\hat{\omega}^2}. \tag{152}$$

Relation (103) in that case is

$$A_n = I_0 e^{\pm i\rho(\hat{\omega})(-(n+1)\Delta_1 + y)} + O\left(\sigma_{1,2}^{-}\right). \tag{153}$$

For the $\hat{\omega} < \kappa$ case, we obtain asymptotic approximations, which contains the first order pole (137). For even *k*

$$\beta_k = -2(W-1)e^{-\rho(n+1-k)\Delta_1} + O\left(\sigma_{1,2}^{+}\right), \tag{154}$$

$$\gamma_k = (2W-1)e^{-\rho((n+1)\Delta_1 + k\Delta_2)} + O\left(\sigma_{1,2}^{+}\right), \tag{155}$$

$$\xi_k = I_0 \left[\begin{array}{l} -(2W-1)e^{\rho(k\Delta_1 - y)} \\ + \dfrac{\left(1-e^{-\rho 2\Delta_1}\right)e^{-\rho(k-2)\Delta_1} - \left(1-e^{-\rho 2\Delta_2}\right)e^{-\rho(k-2)\Delta_2}}{\left(e^{-\rho 2\Delta_1} - e^{-\rho 2\Delta_2}\right)} e^{\rho(k-2)(\Delta_1+\Delta_2)} e^{\rho y} \end{array} \right] + O\left(\sigma_{1,2}^{+}\right), \tag{156}$$

$$\eta_k = I_0 \left[2We^{-\rho(k\Delta_2 + y)} + \frac{\left(1-e^{\rho 2\Delta_2}\right)\left(e^{-\rho k\Delta_1} - e^{-\rho k\Delta_2}\right)}{\left(e^{-\rho 2\Delta_1} - e^{-\rho 2\Delta_2}\right)} e^{-\rho 2(\Delta_1+\Delta_2)} e^{\rho y} \right] + O\left(\sigma_{1,2}^{+}\right), \tag{157}$$

for odd *k*

$$\beta_k = \frac{\left(e^{-\rho(n+2-k)\Delta_1} - e^{-\rho(n+2-k)\Delta_2}\right) - e^{-\rho 2(\Delta_1+\Delta_2)}\left(e^{-\rho(n-k)\Delta_1} - e^{-\rho(n-k)\Delta_2}\right)}{\left(e^{-\rho 2\Delta_1} - e^{-\rho 2\Delta_2}\right)} + O\left(\sigma_{1,2}^{+}\right)^2, \tag{158}$$

$$\gamma_k = -\frac{\left(1-e^{-\rho 2\Delta_1}\right)\left(e^{-\rho(n-k)\Delta_1} - e^{-\rho(n-k)\Delta_2}\right)}{\left(e^{-\rho 2\Delta_1} - e^{-\rho 2\Delta_2}\right)} e^{-\rho(k+1)(\Delta_1+\Delta_2)} + O\left(\sigma_{1,2}^{+}\right), \tag{159}$$

$$\xi_k = -I_0\left[2We^{\rho(k-1)\Delta_2}e^{\rho y} + \frac{\left(1-e^{-\rho 2\Delta_2}\right)\left(e^{-\rho(k-1)\Delta_1} - e^{-\rho(k-1)\Delta_2}\right)}{\left(e^{-\rho 2\Delta_1} - e^{-\rho 2\Delta_2}\right)} e^{\rho(k-1)(\Delta_1+\Delta_2)}e^{-\rho y}\right] + O\left(\sigma_{1,2}^{+}\right), \tag{160}$$

$$\eta_k = I_0\left[\begin{array}{l}\left(2W-1\right)e^{-\rho(k+1)\Delta_1}e^{\rho y} \\ +\dfrac{\left(1-e^{-\rho 2\Delta_2}\right)e^{-\rho(k+1)\Delta_1} - \left(1-e^{-\rho 2\Delta_1}\right)e^{-\rho(k+1)\Delta_2}}{\left(e^{-\rho 2\Delta_1} - e^{-\rho 2\Delta_2}\right)} e^{-\rho y}\end{array}\right] + O\left(\sigma_{1,2}^{+}\right), \tag{161}$$

where $\rho = \rho(\hat{\omega})$ (134), the value (85) is

$$I_0 = -\frac{\rho(\hat{\omega})}{2\hat{\omega}^2}. \tag{162}$$

Relation (103), in that case, is

$$A_n = -I_0 e^{\rho(\hat{\omega})((n+1)\Delta_1 - y)} + O\left(\sigma_{1,2}^{+}\right). \tag{163}$$

4. Results

4.a. *p*-polarization case

The coefficients (96) and (97) are obtained in the general case for any z with relations (131), (119) and other ones (see subsection 3.2). We consider the particular case, i.e. the NIM situation, which implies that the coefficients (96) and (97) are expressed with relations (150), (152), (153), (154)-(161), and (163). Thus, we obtain the expressions for the scalar *p*-polarization part of the electric Green's function (90) for every layer. In the *k*-th layer for $\hat{\omega} > \kappa$ with fixed κ

$$G_k^p(x, y, \pm\hat{\omega}, \kappa) = \pm\frac{\rho(\hat{\omega})}{2i\hat{\omega}^2} e^{\pm i\rho(\hat{\omega}).X(x,y,k)}, \tag{164}$$

for $\hat{\omega} < \kappa$

$$G_k^p(x, y, \pm\hat{\omega}, \kappa) = \frac{\rho(\hat{\omega})}{2\hat{\omega}^2} e^{-\rho(\hat{\omega}).X(x,y,k)}, \tag{165}$$

where

$$X(x, y, k) = |k|\Delta_2 - |x-y| \text{ with } k = -(m-1), \ldots -2, 0, 2, \ldots (n-1), \tag{166}$$

$$X(x, y, k) = -|k|\Delta_1 + |x - \Delta_1 + y| \text{ with } k = -m, \ldots -3, -1, 1, 3, \ldots n. \tag{167}$$

The expressions (164) and (165) are continuous on the layer's surfaces at the x_k coordinates, $k = -(m-1), \ldots, n$, and at the y coordinate of the point source. The second one (165) has no singularities despite the fact that the asymptotic approximations (154)-(155), (160), and (161) for the $\hat{\omega} < \kappa$ case contain the first order poles (137). During solving relations (98)-(101), all approximations have the $O\left(\sigma_{1,2}^{+}\right)$ accuracy of the same infinitesimal order with $\sigma_{1,2}^{+}$, where $\sigma_{1,2}^{+} \to 0$ with $z \to \pm\hat{\omega}$. Thus, we can be sure of the accuracy of the expression obtained, and use equal sign instead of asymptotic equivalence sign with $z \to \pm\hat{\omega}$.

One can stress that relations (164) and (165) are composed of one term. Therefore, the reflection terms are absent, and in the NIM situation the reflection of the *p*-polarization part of the electric field is absent, too.

4.b. *s*-polarization case

Let us consider the *s*-polarization case. It can be shown that relations (96), (97), (98)-(102), and (103) are correct for the *s*-polarization case with the substitution of $\sigma_{kl}^{\pm}$ (76) to $\tau_{kl}^{\pm}$ (93), taking into account relation (95). In the NIM situation for $\hat{\omega} > \kappa$, the value $\tau_{1,2}^{-} = \tau_{1,2}^{-}(z,\kappa) \to 0$ with $z \to \pm\hat{\omega}$ and $\tau_{1,2}^{-} = O\left(\sigma_{1,2}^{-}\right)$ according to (94). For $\hat{\omega} < \kappa$, the value $\tau_{1,2}^{+} = \tau_{1,2}^{+}(z,\kappa) \to 0$ with $z \to \pm\hat{\omega}$ and $\tau_{1,2}^{+} = O\left(\sigma_{1,2}^{+}\right)$ according to (94). Then, the solutions of equations (98)-(101) and the value (103) are obtained in the NIM situation for $\hat{\omega} > \kappa$ by relations (150), (151), and (153), where (152) is

$$I_0 = \pm\frac{1}{2i\rho(\hat{\omega})}, \tag{168}$$

for $\hat{\omega} < \kappa$ by relations (154)-(161), and (163), where (162) is

$$I_0 = \frac{1}{2\rho(\hat{\omega})}, \tag{169}$$

and

$$W = \frac{1}{\tau_{1,2}^{+}} = -\frac{1}{\sigma_{1,2}^{+}} \overset{z\to\pm\hat{\omega}}{\sim} -\left(\frac{\rho(\hat{\omega})\Omega}{2\kappa}\right)^2 \frac{1}{(z-\hat{\omega})(z+\hat{\omega})}. \tag{170}$$

Analogously to the *p*-polarization case we obtain the expressions for the scalar *s*-polarization part of the electric Green's function (90) for every layer. In the *k*-th layer for $\hat{\omega} > \kappa$ with fixed κ,

$$G_k^s(x,y,\pm\hat{\omega},\kappa) = \pm\frac{1}{2i\rho(\hat{\omega})} e^{\pm i\rho(\hat{\omega})X(x,y,k)}, \tag{171}$$

for $\hat{\omega} < \kappa$,

$$G_k^s(x,y,\pm\hat{\omega},\kappa) = \frac{1}{2\rho(\hat{\omega})} e^{-\rho(\hat{\omega})X(x,y,k)}, \tag{172}$$

where $X(x,y,k)$ is defined in (166) and (167).

The expressions (171) and (172) differ from (164) and (165) only in the coefficients of exponents. Therefore, they are continuous on the layer's surfaces at the x_k coordinates, $k = -(m-1),\ldots,n$, and at the y coordinate of the point source. The reflection of the *s*-polarization part of the electric field, as well as of the *p*-polarization part, is absent.

4.c. Electric Green's function

Now we return to the scalar representation of the vector *s*- and *p*-polarization parts of the electric Green's function (41) and (42). For $\hat{\omega} > \kappa$ and even *k*, where $k = -(m-1),\ldots -2,0,2,\ldots(n-1)$,

$$\frac{\partial}{\partial x} G_k^p(x,y,\pm\hat{\omega},\kappa) = -\frac{\partial}{\partial y} G_k^p(x,y,\pm\hat{\omega},\kappa) = \mp i\rho(\hat{\omega})\,\mathrm{sign}(x-y) G_k^p(x,y,\pm\hat{\omega},\kappa), \tag{173}$$

for odd *k*, where $k = -m,\ldots -3,-1,1,3,\ldots n$,

$$\frac{\partial}{\partial x} G_k^p(x,y,\pm\hat{\omega},\kappa) = \frac{\partial}{\partial y} G_k^p(x,y,\pm\hat{\omega},\kappa) = \pm i\rho(\hat{\omega})\,\mathrm{sign}(x-\Delta_1+y) G_k^p(x,y,\pm\hat{\omega},\kappa), \tag{174}$$

for $\hat{\omega} < \kappa$ and even *k*, where $k = -(m-1),\ldots -2,0,2,\ldots(n-1)$,

$$\frac{\partial}{\partial x}G_k^p(x,y,\pm\hat{\omega},\kappa)=-\frac{\partial}{\partial y}G_k^p(x,y,\pm\hat{\omega},\kappa)=\rho(\hat{\omega})\,\mathrm{sign}(x-y)\,G_k^p(x,y,\pm\hat{\omega},\kappa)\,, \tag{175}$$

for odd k, where $k=-m,\ldots-3,-1,1,3,\ldots n$,

$$\frac{\partial}{\partial x}G_k^p(x,y,\pm\hat{\omega},\kappa)=\frac{\partial}{\partial y}G_k^p(x,y,\pm\hat{\omega},\kappa)=-\rho(\hat{\omega})\,\mathrm{sign}(x-\Delta_1+y)\,G_k^p(x,y,\pm\hat{\omega},\kappa)\,, \tag{176}$$

where the sign function is

$$\mathrm{sign}(x)=\begin{cases}+1 & x\geq 0\\ -1 & x<0\end{cases}. \tag{177}$$

Then the electric Green's function (40) from the decomposition (32) in the NIM situation is expressed with the componentwise representation of matrices (37) in the following way: for $\hat{\omega}>\kappa$ in the layers filled with NIM, i.e. with the even index k, where $k=-(m-1),\ldots-2,0,2,\ldots(n-1)$,

$$\mathbf{G}_{\boldsymbol{\kappa}}(x,y,\pm\hat{\omega})=\pm\frac{1}{2i\rho(\hat{\omega})}e^{\pm i\rho(\hat{\omega})(|k|\Delta_2-|x-y|)}\,.$$

$$\cdot\left\{\mathbf{e}_3\times\mathbf{e}_{\boldsymbol{\kappa}}\mathbf{e}_3\times\mathbf{e}_{\boldsymbol{\kappa}}+\frac{\rho^2(\hat{\omega})}{\hat{\omega}^2}\left[\mathbf{e}_{\boldsymbol{\kappa}}\mathbf{e}_{\boldsymbol{\kappa}}\pm\mathrm{sign}(x-y)\frac{\kappa}{\rho(\hat{\omega})}\left(\mathbf{e}_{\boldsymbol{\kappa}}\mathbf{e}_3+\mathbf{e}_3\mathbf{e}_{\boldsymbol{\kappa}}\right)+\frac{\kappa^2}{\rho^2(\hat{\omega})}\mathbf{e}_3\mathbf{e}_3\right]\right\}, \tag{178}$$

in the layers filled with a vacuum, i.e. with the odd index k, where $k=-m,\ldots-3,-1,1,3,\ldots n$,

$$\mathbf{G}_{\boldsymbol{\kappa}}(x,y,\pm\hat{\omega})=\pm\frac{1}{2i\rho(\hat{\omega})}e^{\pm i\rho(\hat{\omega})(-|k|\Delta_1+|x-\Delta_1+y|)}\,.$$

$$\cdot\left\{\mathbf{e}_3\times\mathbf{e}_{\boldsymbol{\kappa}}\mathbf{e}_3\times\mathbf{e}_{\boldsymbol{\kappa}}+\frac{\rho^2(\hat{\omega})}{\hat{\omega}^2}\left[\mathbf{e}_{\boldsymbol{\kappa}}\mathbf{e}_{\boldsymbol{\kappa}}\pm\mathrm{sign}(x-\Delta_1+y)\frac{\kappa}{\rho(\hat{\omega})}\left(\mathbf{e}_{\boldsymbol{\kappa}}\mathbf{e}_3-\mathbf{e}_3\mathbf{e}_{\boldsymbol{\kappa}}\right)-\frac{\kappa^2}{\rho^2(\hat{\omega})}\mathbf{e}_3\mathbf{e}_3\right]\right\}, \tag{179}$$

for $\hat{\omega}<\kappa$ in the layers filled with NIM, i.e. with the even index k, where $k=-(m-1),\ldots-2,0,2,\ldots(n-1)$,

$$\mathbf{G}_{\boldsymbol{\kappa}}(x,y,\pm\hat{\omega})=\frac{1}{2\rho(\hat{\omega})}e^{-\rho(\hat{\omega})(|k|\Delta_2-|x-y|)}\,.$$

$$\cdot\left\{\mathbf{e}_3\times\mathbf{e}_{\boldsymbol{\kappa}}\mathbf{e}_3\times\mathbf{e}_{\boldsymbol{\kappa}}+\frac{\rho^2(\hat{\omega})}{\hat{\omega}^2}\left[\mathbf{e}_{\boldsymbol{\kappa}}\mathbf{e}_{\boldsymbol{\kappa}}-\mathrm{sign}(x-y)\frac{i\kappa}{\rho(\hat{\omega})}\left(\mathbf{e}_{\boldsymbol{\kappa}}\mathbf{e}_3+\mathbf{e}_3\mathbf{e}_{\boldsymbol{\kappa}}\right)-\frac{\kappa^2}{\rho^2(\hat{\omega})}\mathbf{e}_3\mathbf{e}_3\right]\right\}, \tag{180}$$

in the layers filled with a vacuum, i.e. with the odd index k, where $k=-m,\ldots-3,-1,1,3,\ldots n$,

$$\mathbf{G}_{\boldsymbol{\kappa}}(x,y,\pm\hat{\omega})=\frac{1}{2\rho(\hat{\omega})}e^{-\rho(\hat{\omega})(-|k|\Delta_1+|x-\Delta_1+y|)}\,.$$

$$\cdot\left\{\mathbf{e}_3\times\mathbf{e}_{\boldsymbol{\kappa}}\mathbf{e}_3\times\mathbf{e}_{\boldsymbol{\kappa}}+\frac{\rho^2(\hat{\omega})}{\hat{\omega}^2}\left[\mathbf{e}_{\boldsymbol{\kappa}}\mathbf{e}_{\boldsymbol{\kappa}}-\mathrm{sign}(x-\Delta_1+y)\frac{i\kappa}{\rho(\hat{\omega})}\left(\mathbf{e}_{\boldsymbol{\kappa}}\mathbf{e}_3-\mathbf{e}_3\mathbf{e}_{\boldsymbol{\kappa}}\right)+\frac{\kappa^2}{\rho^2(\hat{\omega})}\mathbf{e}_3\mathbf{e}_3\right]\right\}, \tag{181}$$

where the sign function $\mathrm{sign}(x)$ and the function $\rho(\omega)$ is defined in (177) and (134), correspondingly. Thus, with the decompositions (32) and (33), we can obtain the expression of the electric field $\mathbf{E}(\mathbf{x},t)$ for $t\geq t_0$, where $t_0=0$. The electric Green's function can be obtained by the inverse Laplace transform (8) and the Fourier transform (28).

5. Particular case of the layered NIM system

Now, let us consider the particular case of the NIM system studied in the paper (see subsection 2.2). We set $m=1$, where m is the number of layers located to the left side of the zero layer. Then, the system composed of (n+2) parallel alternated layers with numbers $k=-1,0,1,\ldots n$ (Fig. 2). The (−1)-th layer is the

half space unbounded along the direction of the x axis ($x_{-1} = -\infty$) and is empty (vacuum). The point source is located at y coordinate in the (−1)-th layer, i.e. $x_{-1} < y < x_0$ or $-\infty < y < 0$.

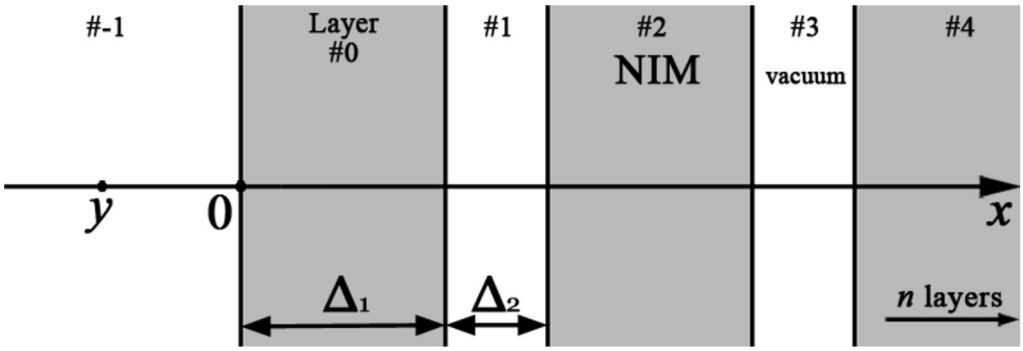

Fig. 2. The NIM system composed of (n+2) parallel layers filled with NIM and vacuum. The point source is located at y coordinate in (−1)-th empty (vacuum) layer that is the half space unbounded along the negative direction of the x axis.

Analogously to the way proposed in the paper (see sections 2-4) we obtain the expressions for the scalar s- and p-polarization parts of the electric Green's function (90) for every layer. In the k-th layer for $\hat{\omega} > \kappa$ with fixed κ,

$$G_k^p(x, y, \pm\hat{\omega}, \kappa) = \pm\frac{\rho(\hat{\omega})}{2i\hat{\omega}^2} e^{\pm i\rho(\hat{\omega})X(x,y,k)}, \tag{182}$$

$$G_k^s(x, y, \pm\hat{\omega}, \kappa) = \pm\frac{1}{2i\rho(\hat{\omega})} e^{\pm i\rho(\hat{\omega})X(x,y,k)}, \tag{183}$$

for $\hat{\omega} < \kappa$,

$$G_k^p(x, y, \pm\hat{\omega}, \kappa) = \frac{\rho(\hat{\omega})}{2\hat{\omega}^2} e^{-\rho(\hat{\omega})X(x,y,k)}, \tag{184}$$

$$G_k^s(x, y, \pm\hat{\omega}, \kappa) = \frac{1}{2\rho(\hat{\omega})} e^{-\rho(\hat{\omega})X(x,y,k)}, \tag{185}$$

where

$$X(x, y, k) = k\Delta_2 - x - y \text{ with } k = 0, 2, \ldots (n-1), \tag{186}$$

$$X(x, y, k) = -(k+1)\Delta_1 + |x - y| \text{ with } k = -1, 1, 3, \ldots n. \tag{187}$$

The expressions in (182), (183), (184), and (185) are the same as in (164), (171), (165), and (172) correspondingly. However, relations (186) and (187) differ from (166) and (167). Nevertheless, the expressions (182)-(185) are continuous on the layer's surfaces at the x_k coordinates, $k = 0, \ldots, (n-1)$, and at the y coordinate of the point source. The reflection of the s- and p-polarization parts of the electric field are absent.

Then the electric Green's function (40) from the decomposition (32) in the NIM situation is expressed with the componentwise representation of matrices (37) in the following way: for $\hat{\omega} > \kappa$ in the layers filled with NIM, i.e. with the even index k, where $k = 0, 2, \ldots (n-1)$,

$$\mathbf{G}_\kappa(x, y, \pm\hat{\omega}) = \pm\frac{1}{2i\rho(\hat{\omega})} e^{\pm i\rho(\hat{\omega})(k\Delta_2 - x - y)} \cdot$$

$$\cdot\left[\mathbf{e}_3 \times \mathbf{e}_\kappa \mathbf{e}_3 \times \mathbf{e}_\kappa + \frac{\rho^2(\hat{\omega})}{\hat{\omega}^2}\left(\mathbf{e}_\kappa \pm \frac{\kappa}{\rho(\hat{\omega})}\mathbf{e}_3\right)\left(\mathbf{e}_\kappa \mp \frac{\kappa}{\rho(\hat{\omega})}\mathbf{e}_3\right)\right], \tag{188}$$

in the layers filled with a vacuum, i.e. with the odd index k, where $k=-1,1,3,\ldots n$,

$$\mathbf{G}_{\kappa}(x,y,\pm\hat{\omega})=\pm\frac{1}{2i\rho(\hat{\omega})}e^{\pm i\rho(\hat{\omega})(-(k+1)\Delta_1+|x-y|)}\cdot$$

$$\cdot\left[\mathbf{e}_3\times\mathbf{e}_{\kappa}\mathbf{e}_3\times\mathbf{e}_{\kappa}+\frac{\rho^2(\hat{\omega})}{\hat{\omega}^2}\left(\mathbf{e}_{\kappa}\mp\mathrm{sign}(x-y)\frac{\kappa}{\rho(\hat{\omega})}\mathbf{e}_3\right)\left(\mathbf{e}_{\kappa}\mp\mathrm{sign}(x-y)\frac{\kappa}{\rho(\hat{\omega})}\mathbf{e}_3\right)\right], \tag{189}$$

for $\hat{\omega}<\kappa$ in the layers filled with NIM, i.e. with the even index k, where $k=0,2,\ldots(n-1)$,

$$\mathbf{G}_{\kappa}(x,y,\pm\hat{\omega})=\frac{1}{2\rho(\hat{\omega})}e^{-\rho(\hat{\omega})(k\Delta_2-x-y)}\cdot$$

$$\cdot\left[\mathbf{e}_3\times\mathbf{e}_{\kappa}\mathbf{e}_3\times\mathbf{e}_{\kappa}+\frac{\rho^2(\hat{\omega})}{\hat{\omega}^2}\left(\mathbf{e}_{\kappa}-\frac{i\kappa}{\rho(\hat{\omega})}\mathbf{e}_3\right)\left(\mathbf{e}_{\kappa}+\frac{i\kappa}{\rho(\hat{\omega})}\mathbf{e}_3\right)\right], \tag{190}$$

in the layers filled with a vacuum, i.e. with the odd index k, where $k=-1,1,3,\ldots n$,

$$\mathbf{G}_{\kappa}(x,y,\pm\hat{\omega})=\frac{1}{2\rho(\hat{\omega})}e^{-\rho(\hat{\omega})(-(k+1)\Delta_1+|x-y|)}\cdot$$

$$\cdot\left[\mathbf{e}_3\times\mathbf{e}_{\kappa}\mathbf{e}_3\times\mathbf{e}_{\kappa}+\frac{\rho^2(\hat{\omega})}{\hat{\omega}^2}\left(\mathbf{e}_{\kappa}+\mathrm{sign}(x-y)\frac{i\kappa}{\rho(\hat{\omega})}\mathbf{e}_3\right)\left(\mathbf{e}_{\kappa}+\mathrm{sign}(x-y)\frac{i\kappa}{\rho(\hat{\omega})}\mathbf{e}_3\right)\right], \tag{191}$$

where the sign function $\mathrm{sign}(x)$ and the function $\rho(\omega)$ is defined in (177) and (134), correspondingly. Thus, with the decompositions (32) and (33), we can obtain the expression of the electric field $\mathbf{E}(\mathbf{x},t)$ for $t \geq t_0$, where $t_0 = 0$. The electric Green's function can be obtained by the inverse Laplace transform (8) and the Fourier transform (28).

6. Conclusions

In this paper, we solved the problem of obtaining the electric Green's function for the layered NIM system. The point source is located inside a certain NIM layer of the system. We proposed the way of its obtaining for any frequency ω. In the NIM situation, we obtained relations for the Fourier transformed electric Green's function, which is the kernel of the decomposition (32). Then, with the decomposition (32) and the function of the initial field's configuration, the electric field's values at any point of the system can easily be evaluated. The magnetic field can as well be obtained with reasoning similar to the presented above. The obtained formulae are symmetric, relative to the position of the point source. This fact shows the correlation with the physical conception of the electromagnetic field propagated into the system composed of isotropic homogeneous layers. Besides, the reflection in the NIM situation is absent.

For the particular case of considered NIM system, where the point source is located in the last-to-left-side unbounded empty (vacuum) layer, we obtained the similar formulae for the electric Green's function and observed that the reflection in the NIM situation also is absent.

We did not consider absorption in the NIM case. This is straightforward to do along the same lines. The two frequencies $\pm\hat{\omega}$ now acquire a negative imaginary part, so the reflection term in Green's function no longer vanishes.

We present formulae for the NIM system under some particular non-essential limitations (The number of layers is odd and can be 7, 11, 15, etc; the point source is located in the layer filled with NIM; the number of layers located to the left and to the right from the point source, is odd and greater than or equal to three, so at least seven layers can form the system). However, the corresponding formulae for other cases could be evaluated by the same way.

The system can be composed of arbitrary finite number of layers. This fact allows us to use the considered system as a model for simulation or engineering of the real objects, such as superlens systems and

multilayer NIM coverings. Obtained absence of reflection (for the leading asymptotic term near the NIM frequency) opens an intriguing prospect.

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