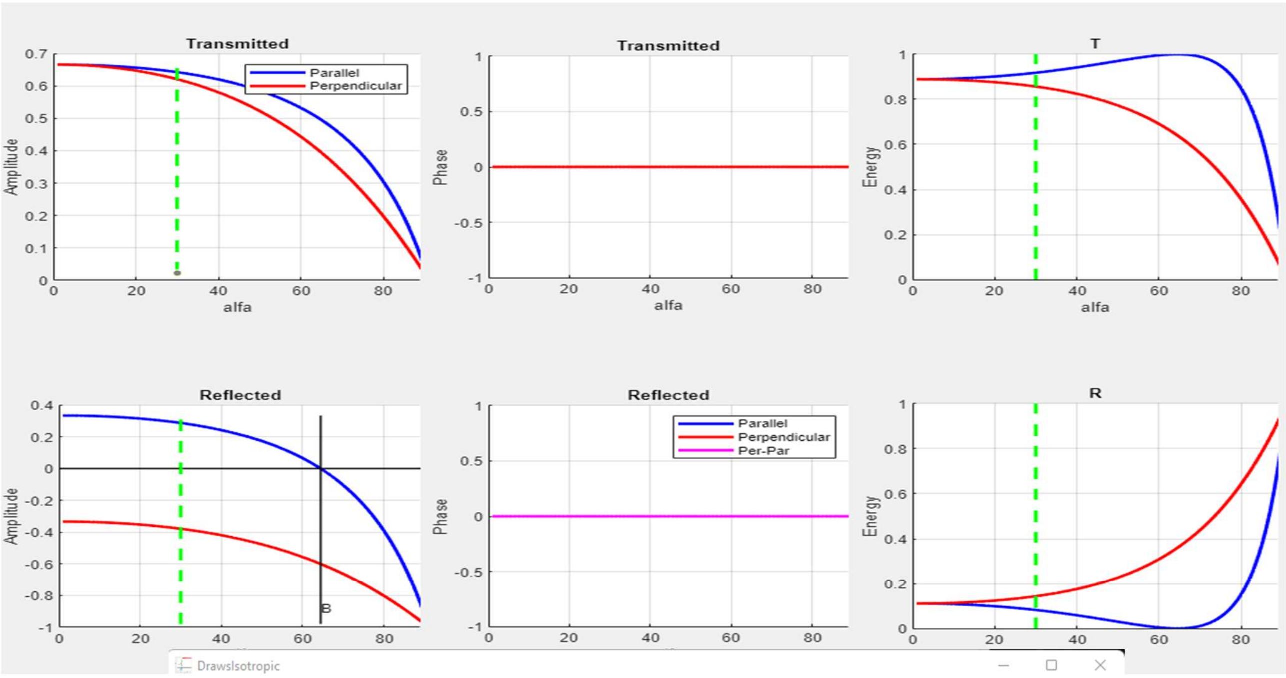
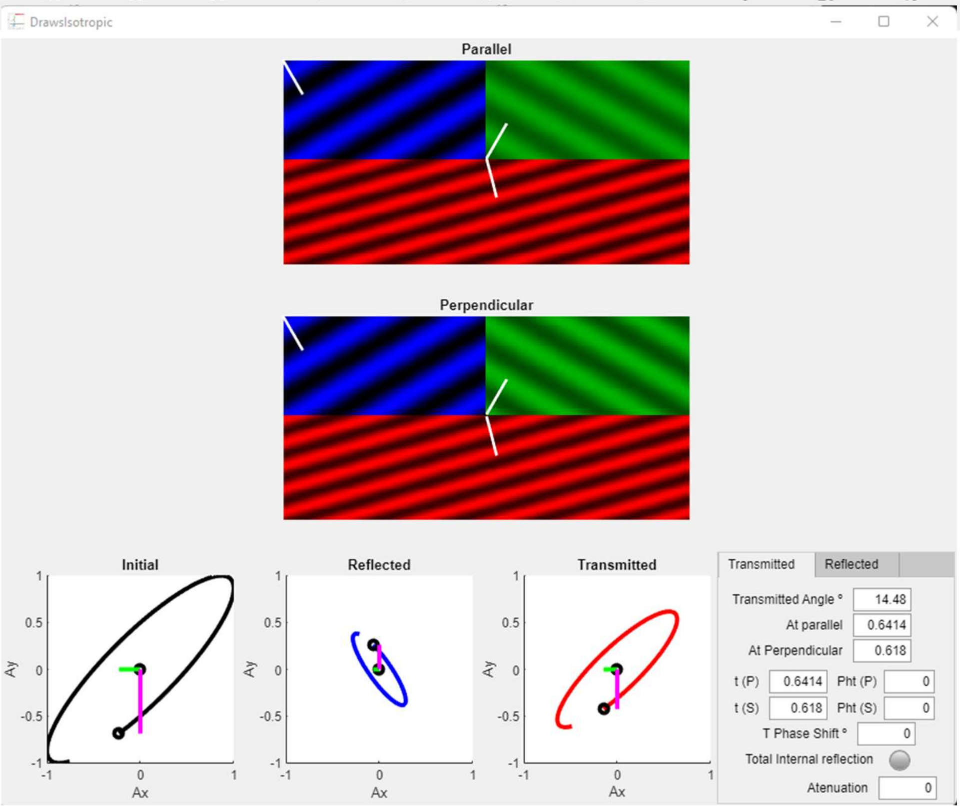
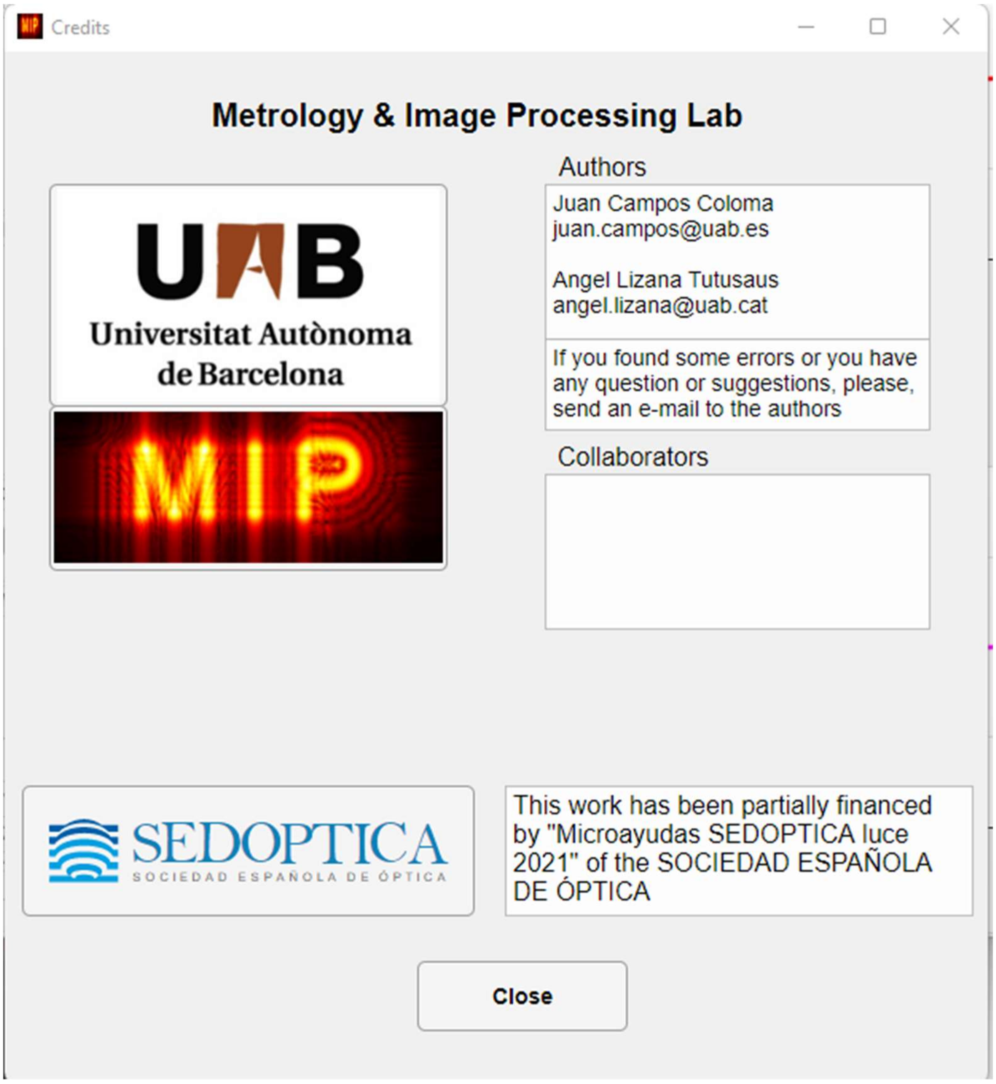
ISOTROPIC MEDIA







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Bellaterra, 29/06/2022

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2 THEORY SUMMARY

When the light impinges on the separation surface between two media, part is reflected and part is refracted.

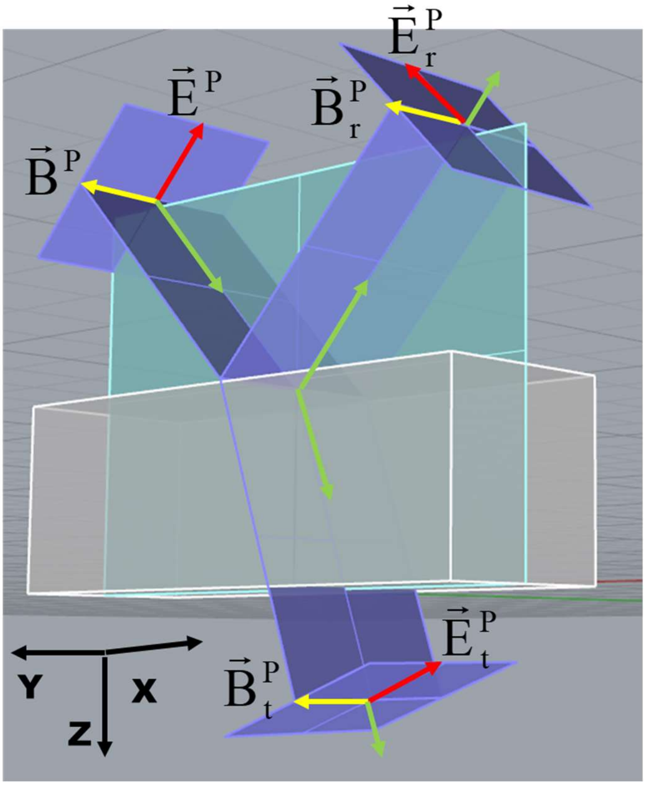


Figure 2-1.Sign convention when the electric field is parallel to the plane of incidence. This is denoted P wave

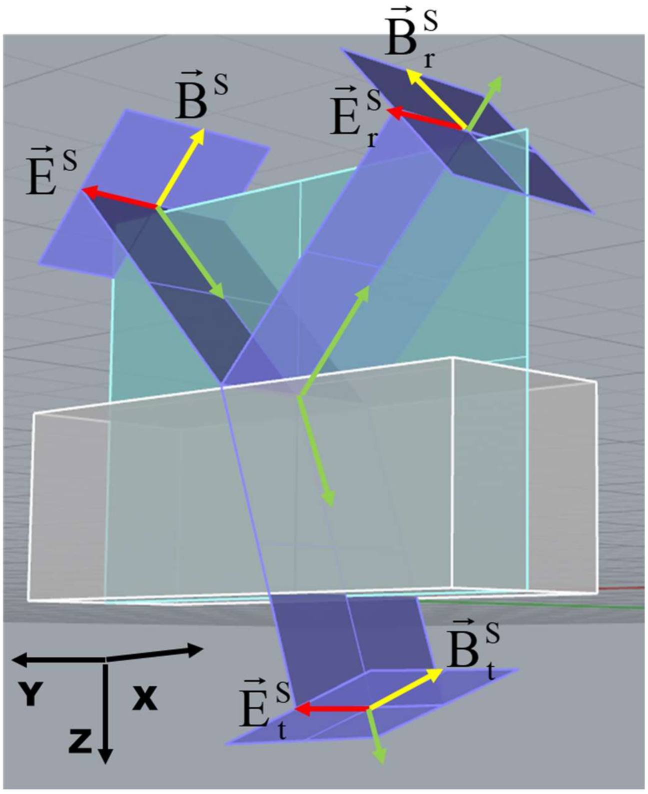


Figure 2-2. Sign convention when the electric field is perpendicular to the plane of incidence. This is denoted S wave

The propagation direction of the incident light and the normal to the interface define a plane, which is referred to as the plane of incidence. Two polarizations of particular interest are the E field polarized in the plane of incidence (P wave), and E field polarized perpendicular to the plane of incidence (S wave). The sign convention for the expressions below is shown in Figure 2-1 and Figure 2-2. The incidence plane is the XZ plane

We assume a monochromatic plane wave as the incident light that is expressed as

$$\vec{E} = \vec{A}\,\mathrm{e}^{i\left(\omega\,\mathrm{t}\text{-}\mathrm{k}\,\vec{\mathrm{r}}.\vec{s}\right)} = \vec{A}\,\mathrm{e}^{i\left(\omega\,\mathrm{t}\text{-}\vec{\mathrm{k}}.\vec{\mathrm{r}}\right)} \tag{1.1}$$

with

$$k^2 = \mu\varepsilon\ \omega^2 \Rightarrow k = \sqrt{\mu\varepsilon}\ \omega = \frac{n}{c}\omega \tag{1.2}$$

$$k_x = k\sin(\theta) = \mathrm{k} = \sqrt{\mu\varepsilon}\ \omega\sin(\theta) = \frac{n\omega}{\mathrm{c}}\sin(\theta) \tag{1.3}$$

Then, the refracted and transmitted beams are expressed as

$$\vec{E}_r = \vec{A}_r \ \mathrm{e}^{i(\omega \ \mathrm{t\text{-}k} \ \ \vec{\mathrm{r}}.\vec{s}_r)} = \vec{A}_r \ \mathrm{e}^{i(\omega \ \mathrm{t\text{-}}\vec{\mathrm{k}}_r.\vec{\mathrm{r}})}$$
$$\vec{E}_t = \vec{A}_t \ \mathrm{e}^{i(\omega \ \mathrm{t\text{-}}\tilde{\mathrm{k}}_t \ \vec{\mathrm{r}}.\vec{s}_t)} = \vec{A}_t \ \mathrm{e}^{i(\omega \ \mathrm{t\text{-}}\tilde{\vec{\mathrm{k}}}_t.\vec{\mathrm{r}})} \tag{1.4}$$

were $\tilde{\mathrm{k}}$ is, in general, complex

$$\tilde{k}_t^{\ 2} = \mu_t \varepsilon_t \ \omega^2 \Rightarrow k_t = \sqrt{\mu_t \varepsilon_t} \ \omega = \frac{\tilde{n}_t}{c}\omega = \frac{n_t - i\kappa}{c}\omega \tag{1.5}$$

The X component of the k vectors is conserved

$$\tilde{k}_x^t = k_{\ x}^{\ t} + ia_{\ x}^{\ t} = k_x \Rightarrow \begin{cases} k_{\ x}^{\ t} = k_x \\ a_{\ x}^{\ t} = 0 \end{cases} \tag{1.6}$$

The Z components are:

$$k_z = \sqrt{k^2 - k_x^2}$$
$$\tilde{k}_{t\,z} = \sqrt{\tilde{k}t^2 - \tilde{k}_{t\,x}^2} = \sqrt{\tilde{k}_t^{\ 2} - k_x^2} \tag{1.7}$$

Then, the parallel and perpendicular components of the elctric fields are given by

(P wave) Parallel component

$$E_r^P = r^P E^P \ \mathrm{e}^{i(\omega \ \mathrm{t\text{-}}(xk_x - zk_z))}$$
$$E_t^P = t^P E^P \ \mathrm{e}^{i(\omega \ \mathrm{t\text{-}}(xk_x + z\tilde{k}_{t\,z}))} \tag{1.8}$$

(S wave) Perpendicular component

$$E_r^S = r^S E^S \ \mathrm{e}^{i(\omega \ \mathrm{t\text{-}}(xk_x - zk_z))}$$
$$E_t^P = t^S E^S \ \mathrm{e}^{i(\omega \ \mathrm{t\text{-}}(xk_x + z\tilde{k}_z))} \tag{1.9}$$

r^P, t^P, r^S, t^S are the Fresnel coefficients that are given by

$$\boxed{t^{P} = \frac{2\varepsilon_t k_Z}{\varepsilon_t k_Z + \varepsilon k_{tZ}} \sqrt{\frac{\varepsilon \mu_t}{\varepsilon_t \mu}} \quad ; \quad r^{P} = \frac{\varepsilon_t k_Z - \varepsilon k_{tZ}}{\varepsilon_t k_Z + \varepsilon k_{tZ}}} \tag{1.10}$$

$$\boxed{t^{S} = \frac{2\mu_t k_z}{\mu k_{t\,z} + \mu_t k_z} \quad ; \quad r^{S} = \frac{\mu_t k_z - \mu k_{t\,z}}{\mu k_{t\,z} + \mu_t k_z}} \tag{1.11}$$

3 USER INTERFACE.

3.1 FRESNEL COEFFICIENTS

Initially the Application shows two windows.

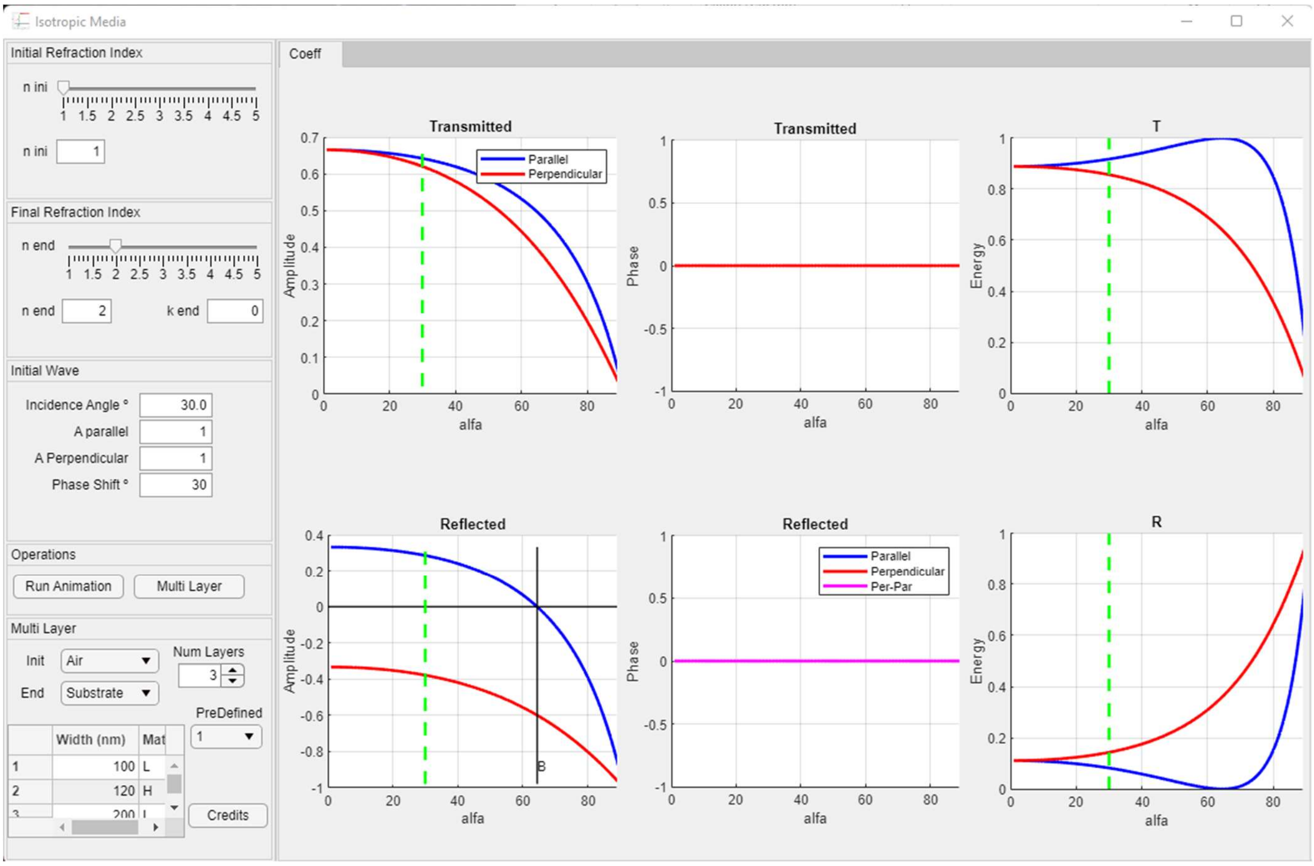


Figure 3-1. Control window. Data can be changed, and the results of the Fresnel Coefficients are shown. nini = 1; nedn(2,i0)

On the left of this figure the controls to change the parameters are showm. The initial refraction index is real (perfect dielectric. The final refraction index is, in general complex. The real part “n” and the imaginary part “k” can be entered

On the “Initial Wave” panel the following parameters can be introduced: Incidence angle; the amplitudes of the parallel (P) and perpendicular (S) components; the Phase difference between S and P components (PhS – PhP).

On the right part of the windows the Fresnel coefficients and the phase differences are shown

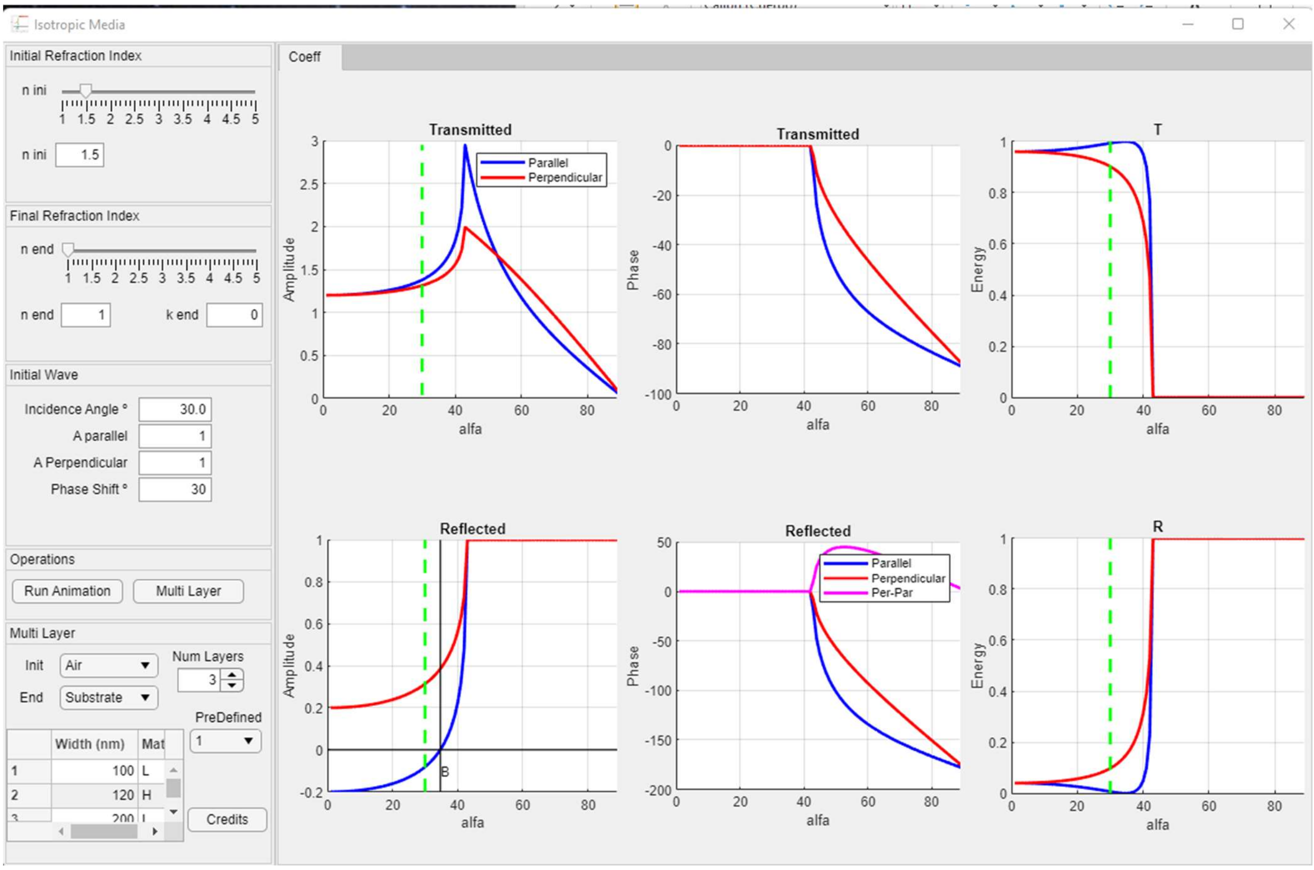


Figure 3-2. Fresnel coefficients when nini = 1.5; nedn(1,i0)

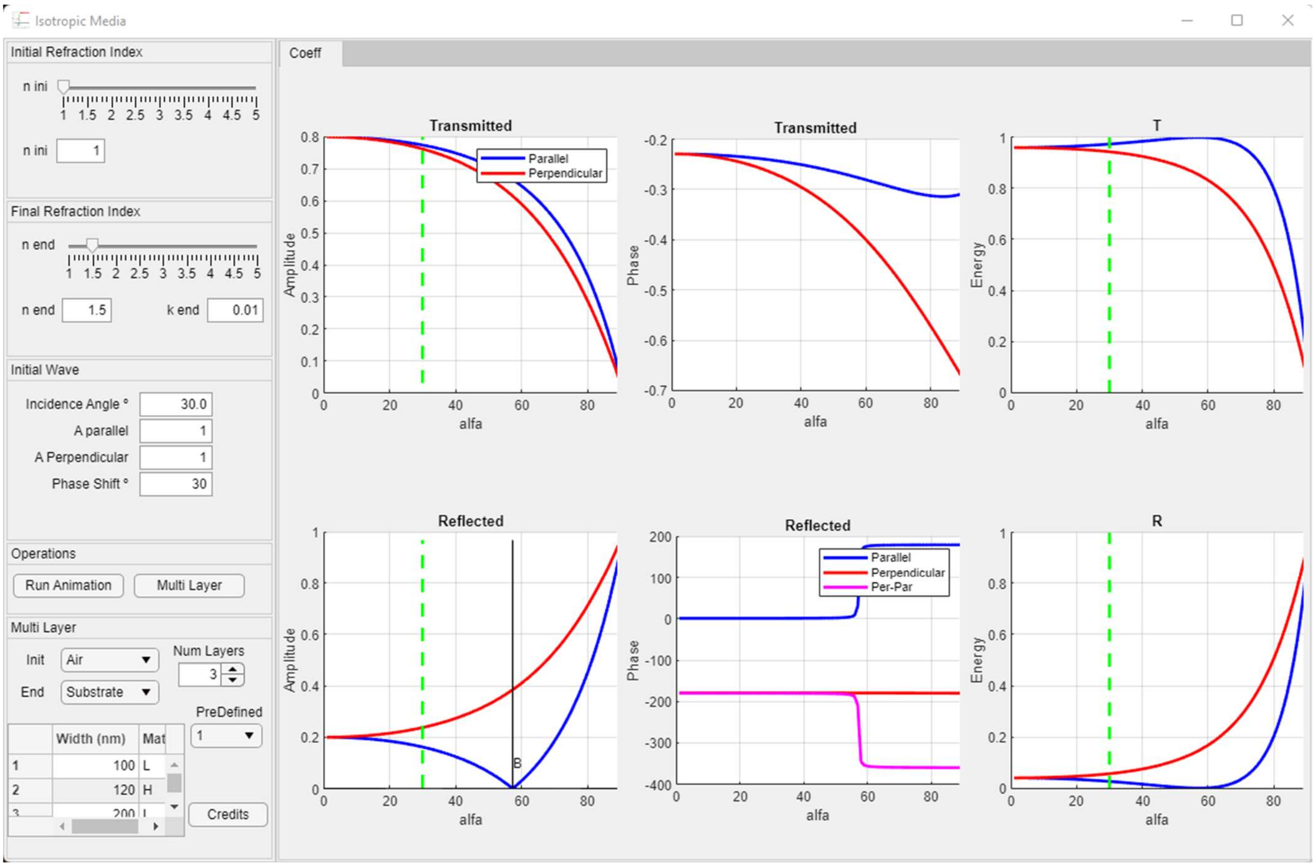


Figure 3-3 . Fresnel coefficients when nini = 1; nend(1.5,i0.01)

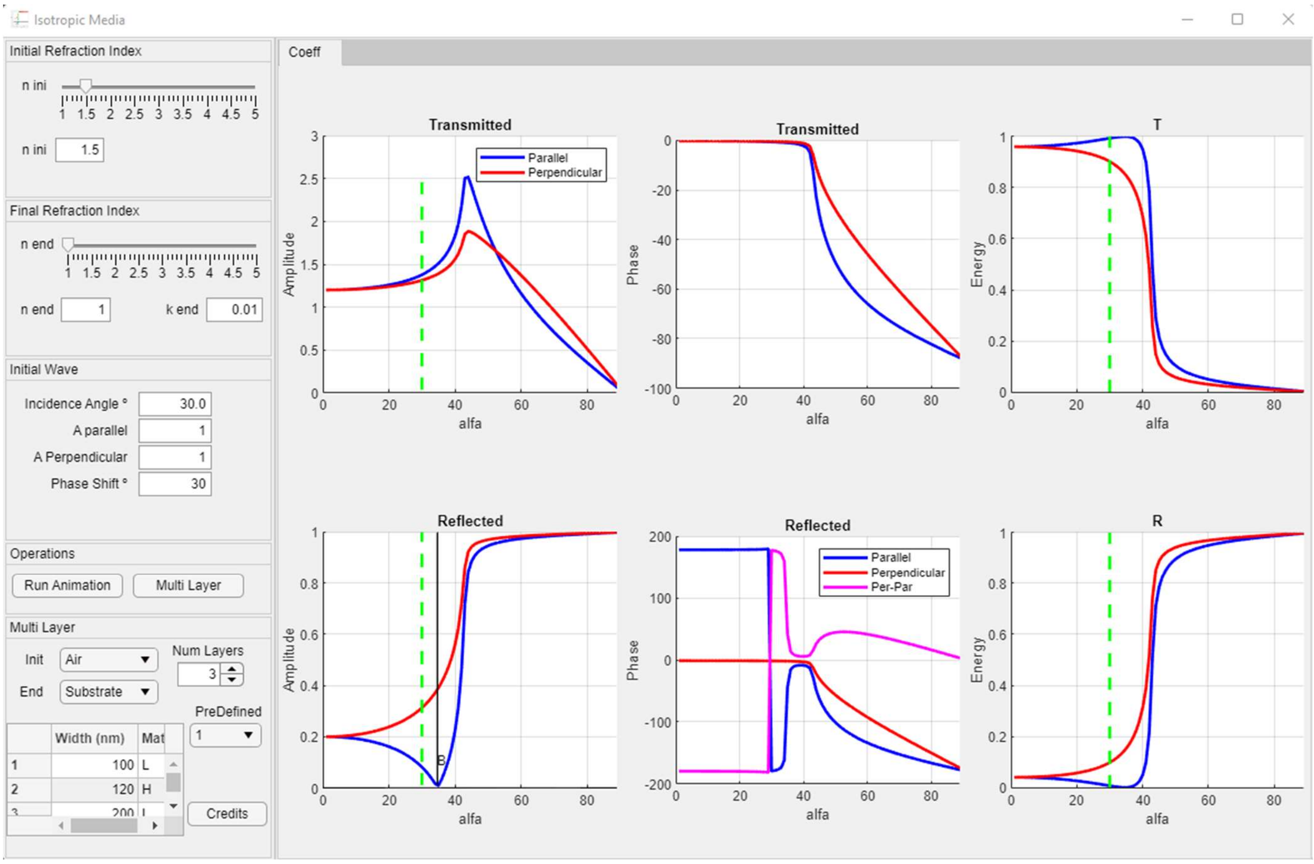


Figure 3-4. . Fresnel coefficients when nini = 1.5; nedn(1,i0.01)

3.2 Animation Window

xThe second window that appears is the animation window. By pressing the button “Stop Animation”/”Run Animation” the window with the animations is displayed or hidden.

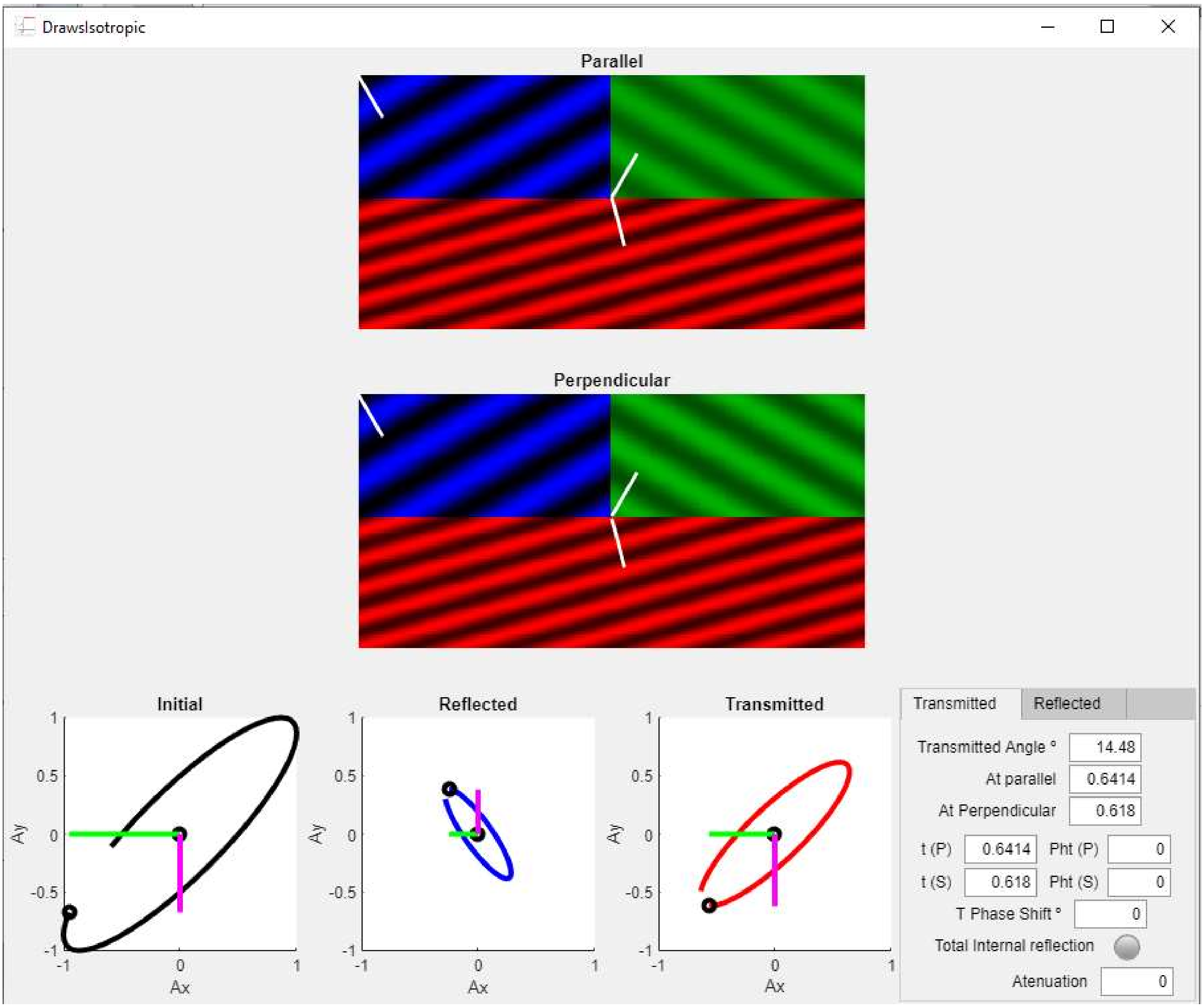


Figure 3-5. Animation window. Top/Center: P and S components of the incident (Blue) reflected (green) and transmitted (red) waves. Bottom: Polarization ellipses of the initial, reflected, and transmitted waves. Bottom Right parameters of the transmitted and reflected light. nini = 1.5; nedn(1,i0). Angle of incidence = 30º

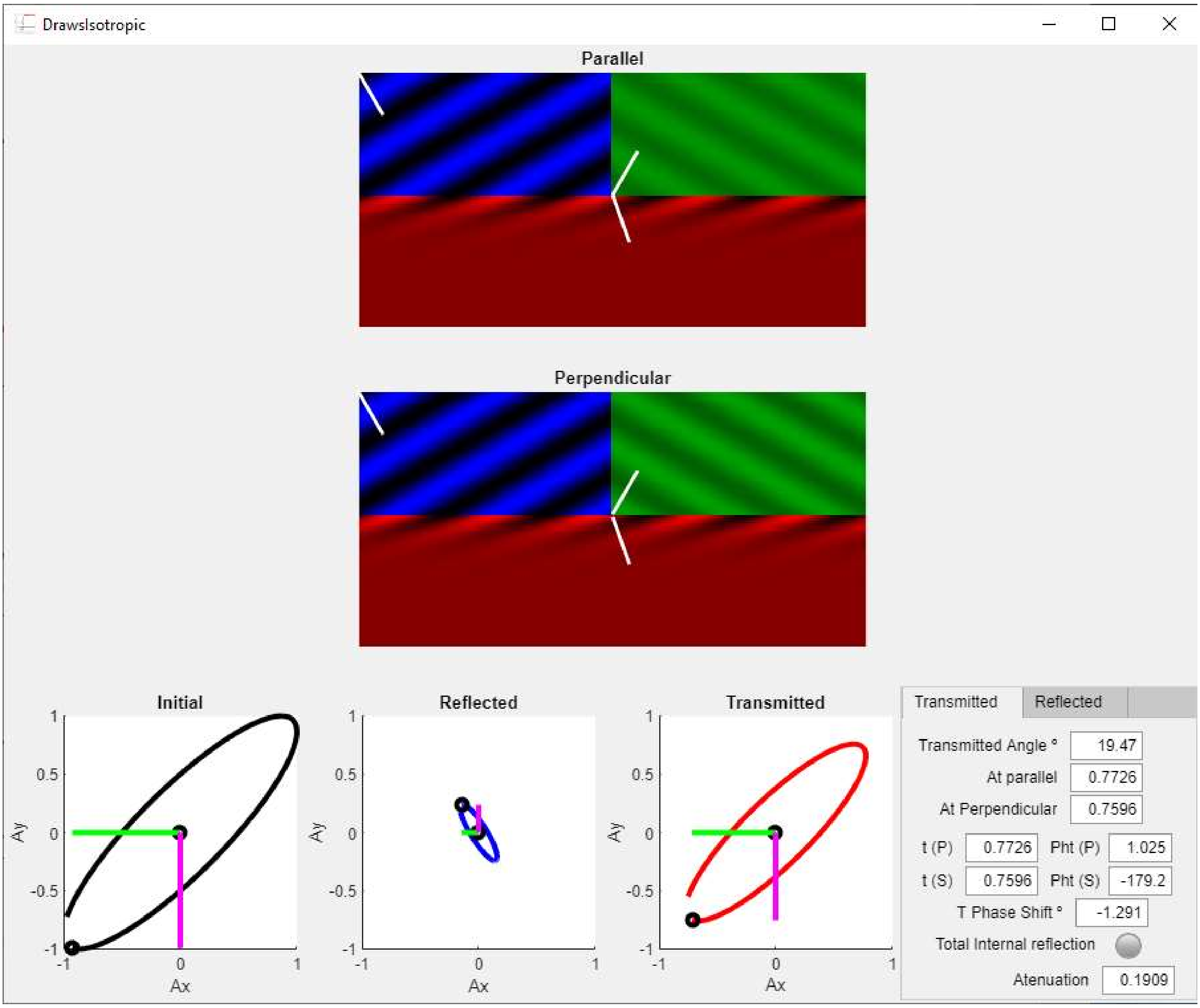


Figure 3-6. nini = 1; nedn(1.5,i0.01)

3.3 MULTILAYER ENERGY TRANSMISSION. [1], [2]

Let us define the complex refractive index as

$$\tilde{n} = n - ik \tag{1.12}$$

The characteristic admittance of a material is given by

$$y = (n - ik)\eta_{free}$$

Were

$$\eta_{free} = \sqrt{\frac{\varepsilon_0}{\mu_0}} = 2.6544x10^{-3}\,S \tag{1.13}$$

The relation between the electric and magnetic field for harmonic plane waves is given by

$$H = yE \tag{1.14}$$

Let us call the amplitude reflection coefficient ρ and the amplitude transmission coefficient τ as

$$\rho = \frac{\text{reflected amplitude(tangential Component)}}{\text{incident amplitude(tangential Component)}} = \frac{\eta_0 - Y}{\eta_0 + Y}$$

(1.15)

$$\tau = \frac{\text{transmitted amplitude(tangential Component)}}{\text{incident amplitude(tangential Component)}} = \frac{2\eta_0}{\eta_0 + Y}$$

Expression (1.15) is valid for normal incidence, were η_0 is the surface admittance for the incident medium and Y is the surface admittance of the multilayer and substrate, that is given by

$$Y = \frac{C}{B} \tag{1.16}$$

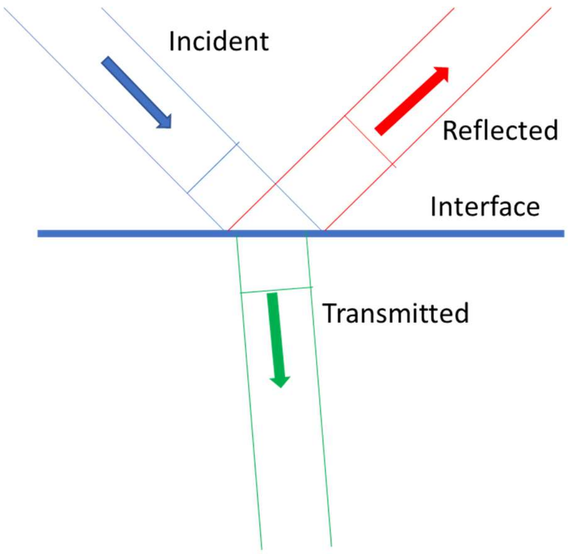
Were

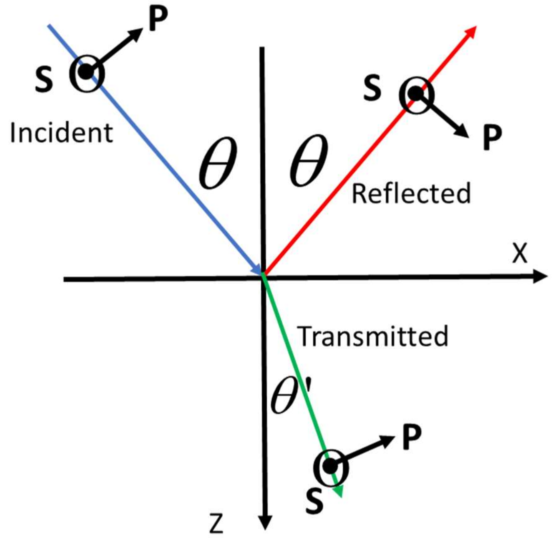
$$\begin{bmatrix} B \\ C \end{bmatrix} = \left\{ \prod_{r=1}^{q} \begin{bmatrix} \cos(\delta_r) & i\sin(\delta_r)/\eta_r \\ i\sin(\delta_r)\eta_r & \cos(\delta_r) \end{bmatrix} \right\} \begin{bmatrix} 1 \\ \eta_s \end{bmatrix} \tag{1.17}$$

Were

- B and C are normalized tangential electric and magnetic fields at the input surface
- $\delta_r = \frac{2\pi}{\lambda} d_r \tilde{n}_r$ is the phase thickness of layer r
- $\tilde{n}_r = n_r - ik_r$ is the complex refractive index of layer r
- d_r is the physical thickness of layer r
- q is the number of layers, being q=1 next to the incident medium

- η_s is the characteristic admittance of the substrate.





Let us define the reflectance, R, as the ratio of the irradiance of the reflected beam to that of the incident beam and transmittance, T, as the ratio of the irradiance of the transmitted beam to that of the incident beam, as shown in the figure. Note that the cross-sectional area is the same for the three beams, but they subtend different area at the boundary

$$R = \rho\rho^* = \left(\frac{\eta_0 - Y}{\eta_0 + Y}\right)\left(\frac{\eta_0 - Y}{\eta_0 + Y}\right)^* \qquad (1.18)$$
$$T = \frac{4\eta_0 \operatorname{Re}(Y)}{(\eta_0 + Y)(\eta_0 + Y)^*}$$

For oblique incidence we need to separate the fields in two components one parallel to the incident plane (p (TM) polarized) and the other perpendicular to the incident plane (s (TE) polarized)

Note that in thin film the tangential components are used to define the reflection and transmission coefficients. This is different from the Fresnel coefficients which uses the total electric and magnetic fields of the waves. However, the differences are confined to the amplitude transmission coefficient for p-polarized light.

The expressions given above are still valid with the following changes:

Optical admittance for s-polarization:

$$\eta_s = y\cos(\theta) \qquad (1.19)$$

Optical admittance for p-polarization:

$$\eta_p = y/\cos(\theta) \qquad (1.20)$$

Phase thickness

$$\delta_r = \frac{2\pi}{\lambda} d_r \tilde{n}_r \cos(\theta_r) = \frac{2\pi}{\lambda} d_r \sqrt{n_r^2 - k_r^2 - n_0^2 \sin^2(\theta_0) - 2i n_r k_r} \qquad (1.21)$$

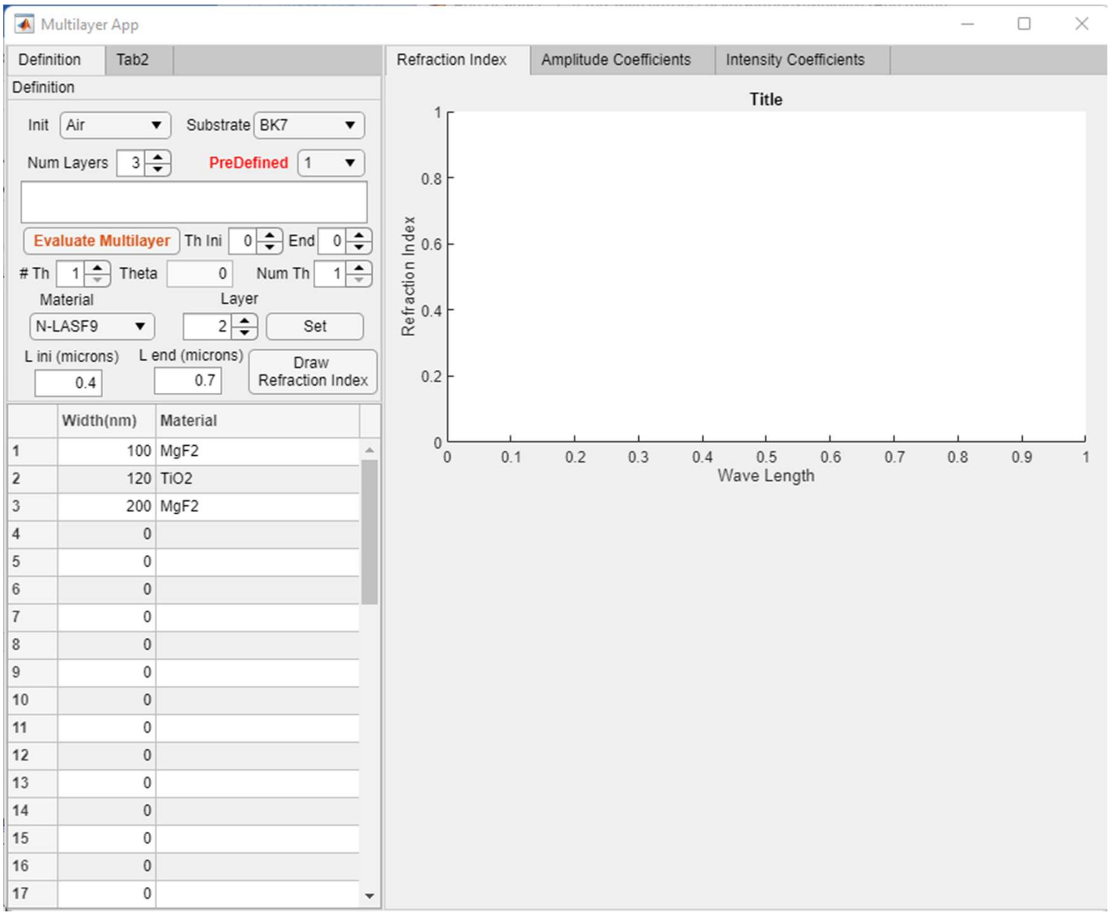
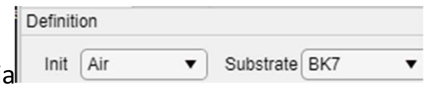


Figure 7. Multilayer App Main Window

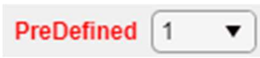
The main window for multilayer application is shown in Figure 7. As usual, on the left, the controls to give the parameters are shown, and on the right some results are displayed.

To define the multilayer, we need to provide the Initial and substrate media 

And the number of layers 

	Width(nm)	Material
1	100	MgF2
2	120	TiO2
3	200	MgF2
4	0	
5	0	
6	0	

The width in nanometers and the material of each layer should also be defined. There are some predefined structures



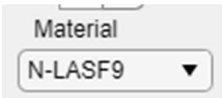
There are some predefined multilayers 1-4, or the data can be read from an excel file

The structure of the excel file is as follow:

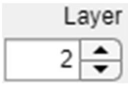
- In the first line a description of the system
- The second row gives the number of layers

Multicapa 4	
11	
Air	Inf
TiO2	60
MgF2	100
TiO2	60
MgF2	100
TiO2	60
MgF2	200
TiO2	60
MgF2	100
TiO2	60
MgF2	100
TiO2	60
BK7	Inf

- The next rows describe the materials and thicknesses. The third row corresponds to the initial media, and the last row corresponds to the substrate. The thicknesses are given in nanometers.

- You can choose any material given in 

You can define manually a new system by introducing the number of layers and after gibing the thickness and the material of each layer. The steps are:

- Select the Init Media and the substrate
- Select the number of layers >=50
- For each layer
  - Select the layer you want to define 

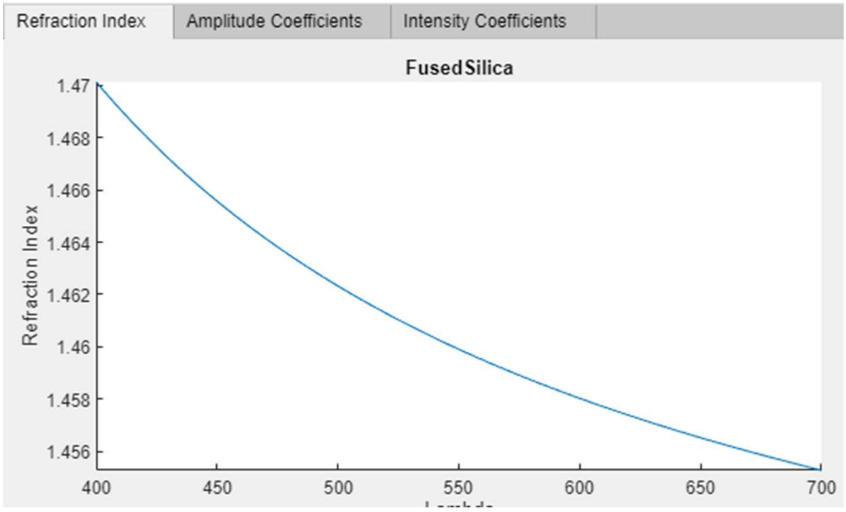
  - Select the material 

  - Write the Width(nm) in the table
  - Press the button 

You can visualize the dependence with the wavelegths of the refraction index of the material selected in 

for that you have to select the wavelength range 

and press the button "Draw Refraction Index". The refraction index appears in



Once you have defined the multilayer you can calculate the amplitude and reflection coefficients

- Select the Theta range and Number of Theta (incidence angle) you want to evaluate, as well the range of wavelengths.
- Pres the button "Evaluate multilayer"

- If you have selected more than one incidence angle, initially, the displays show the first angle
- To change the angle to visualize, you have to select the number of the angle 

The corresponding Theta is displayed

The amplitude coefficients (absolute values and phases) for the S and P waves and for the reflected and transmitted waves are shown in

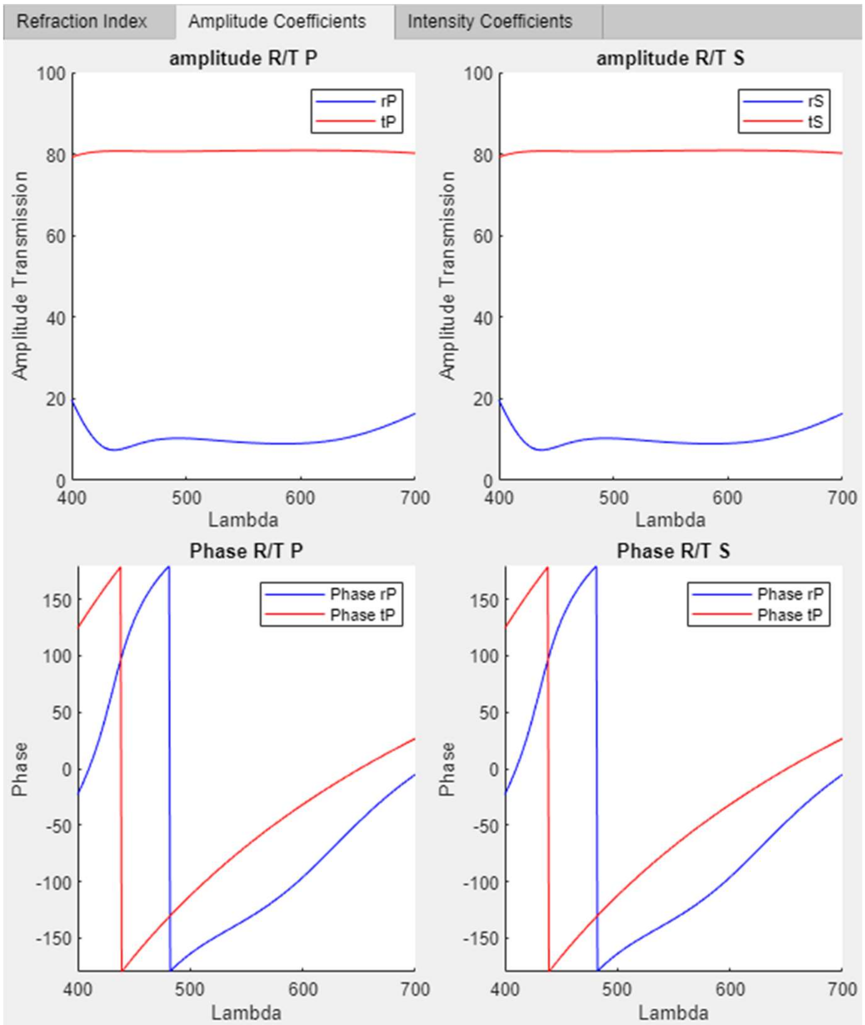


Figure 8 Amplitude coefficients for the first predefined multilayer

And the irradiance coefficients in

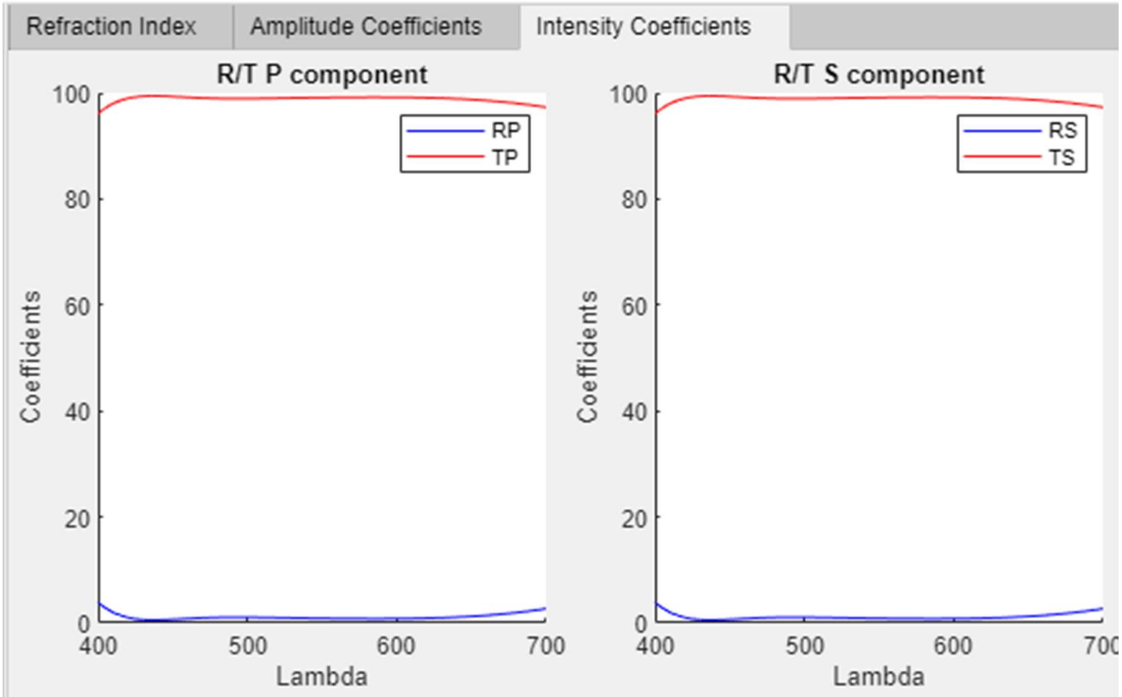


Figure 9 Irradiance coefficients for the first predefined multilayer

4 REFERENCES

[1] H A. Macleod, “Thin Film Calculator Manual.” http://wp.optics.arizona.edu/milster/wp-content/uploads/sites/48/2016/06/Thin-film-calculator-from-Dissertation_JunZhang_080110_optimized.pdf (accessed Jul. 08, 2022).

[2] H. A. Macleod, “Thin-film optical filters,” *Thin-Film Optical Filters, Fourth Edition*, pp. 1–782, Jan. 2010, doi: 10.1201/9781420073034/THIN-FILM-OPTICAL-FILTERS-ANGUS-MACLEOD-ANGUS-MACLEOD.