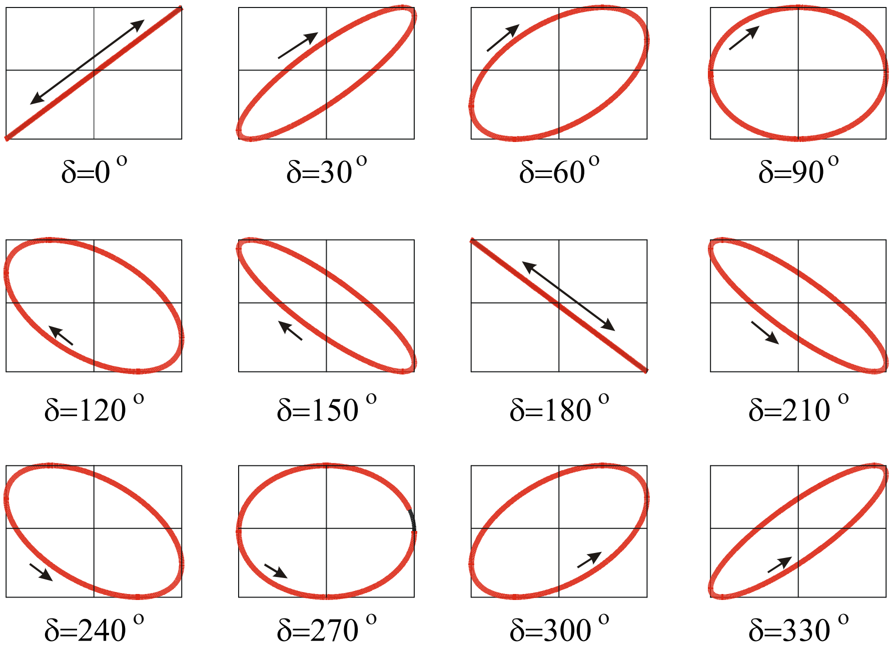
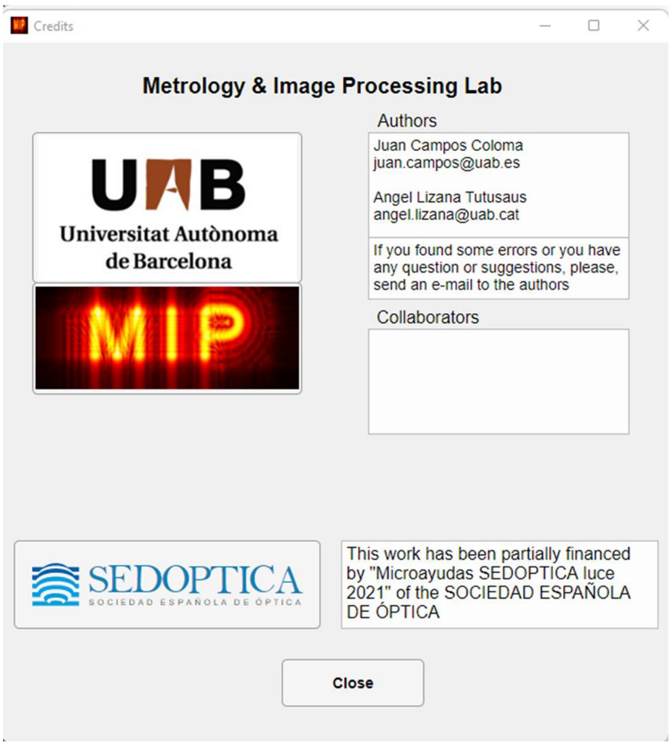
POLARIZED LIGHT





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Bellaterra, 29/06/2022

1 CONTENT

2 PLANE WAVES OVERVIEW IN NON-ABSORBING MEDIA

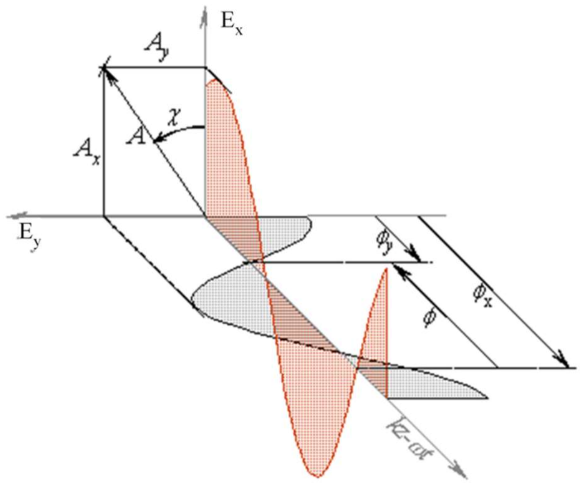
A harmonic plane wave can be written in complex notation as

$$\mathbf{E} = \mathbf{E_0} e^{i(\omega t - \mathbf{k}.\mathbf{r})} \tag{0.1}$$

Were

$$k^2 = \mu \, \varepsilon \, \omega^2 \tag{0.2}$$

3 LIGHT POLARIZATION STATES



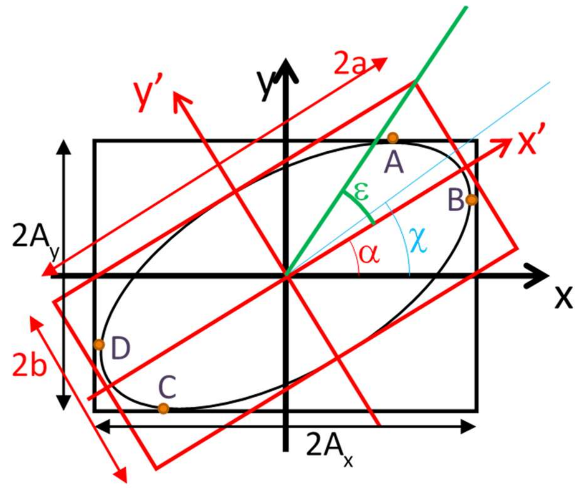
3.1 SUPERPOSITION OF TWO WAVES WITH PERPENDICULAR ELECTRICAL VECTORS

$$E_x(z,t) = A_x e^{i(wt-kz+\varphi_x)} = A_x e^{i\varphi_x} e^{i(wt-kz)}$$
$$E_y(z,t) = A_y e^{i(wt-kz+\varphi_y)} = A_y e^{i\varphi_y} e^{i(wt-kz)} \quad (0.3)$$

Irradiance of an elliptically polarized wave

$$I = < S_z > = \frac{\tilde{n}}{\mu c}\frac{1}{2}\left(A_x^2 + A_y^2\right) \quad (0.4)$$

3.2 POLARIZATION ELLIPSE



$$\frac{E_x^2}{A_x^2} + \frac{E_y^2}{A_y^2} - 2\frac{E_x}{A_x}\frac{E_y}{A_y}\cos(\varphi) = \sin^2(\varphi)$$
$$\varphi = \varphi_y - \varphi_x \quad (0.5)$$

$$\varepsilon \Rightarrow \text{ellipticity} \qquad \tan\varepsilon = \frac{b}{a}$$
$$\alpha \Rightarrow \text{azimuth} \quad (0.6)$$

$$\tan 2\alpha = \frac{2A_x A_y}{A_x^2 - A_y^2}\cos\varphi = \tan 2\chi \cos\varphi \quad (0.7)$$

$$0<\varphi<\pi \Rightarrow \quad \text{RIGHT HANDED}$$
$$\pi<\varphi<2\pi \Rightarrow \quad \text{LEFT HANDED} \quad (0.8)$$

3.3 JONES VECTOR

$$\vec{J} = \begin{pmatrix} A_x\, e^{i\varphi_x} \\ A_y\, e^{i\varphi_y} \end{pmatrix} = e^{i\varphi_x}\begin{pmatrix} A_x \\ A_y\, e^{i\varphi} \end{pmatrix} A e^{i\varphi_x}\begin{pmatrix} \cos\chi \\ \sin\chi\, e^{i\varphi} \end{pmatrix} \quad (0.9)$$

$$A^2 = A_x^2 + A_y^2$$

With
$$\chi = \arctan\frac{A_y}{A_x} \qquad \tan\chi = \frac{A_y}{A_x} \qquad \tan 2\chi = \frac{2A_xA_y}{A_x^2 - A_y^2} \tag{0.10}$$
$$\varphi = \varphi_y - \varphi_x$$

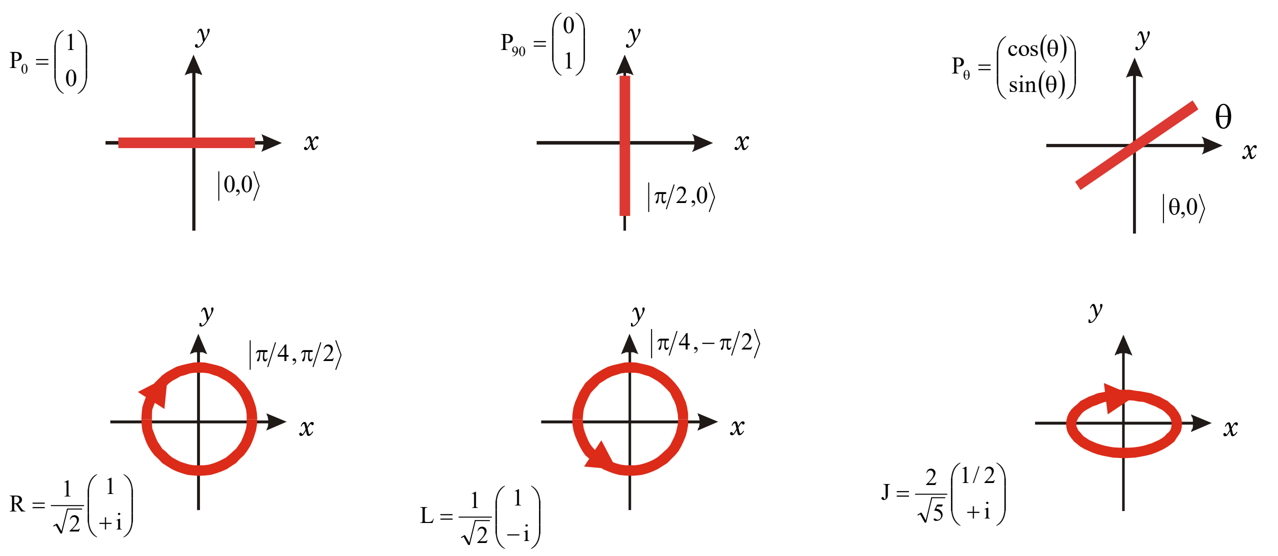


Figure 3-1. Some examples of Jones vector representation

Two Jones vectors are orthogonal if

$$\vec{J}_1^{+} \cdot \vec{J}_2 = 0 \tag{0.11}$$

Decomposition in two orthogonal polarization states

$$\begin{aligned} J &= \begin{pmatrix} A_x \\ A_y e^{i\varphi} \end{pmatrix} = A_x \begin{pmatrix} 1 \\ 0 \end{pmatrix} + A_y e^{i\varphi} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \rightarrow \text{In Linear States} \\ &= \frac{1}{2}\left(A_x - iA_y e^{i\varphi}\right)\begin{pmatrix} 1 \\ +i \end{pmatrix} + \frac{1}{2}\left(A_x + iA_y e^{i\varphi}\right)\begin{pmatrix} 1 \\ -i \end{pmatrix} \rightarrow \text{In Circular States} \end{aligned} \tag{0.12}$$

3.4 Stokes & Mueller Formalism

The Stokes polarization parameters for a plane wave are given by

$$
\begin{aligned}
S_0 &= E_{0x}^2 + E_{0y}^2 \\
S_1 &= E_{0x}^2 - E_{0y}^2 \\
S_2 &= 2E_{0x}E_{0y}\cos\delta \\
S_3 &= 2E_{0x}E_{0y}\sin\delta
\end{aligned} \tag{0.13}
$$

The Stokes parameters enable us to describe the degree of polarization P for any state of polarization. By definition

$$
P = \frac{I_{pol}}{I_{tot}} = \frac{\left(S_1^2 + S_2^2 + S_3^2\right)^{1/2}}{S_0} \qquad 0 \le \mathrm{P} \le 1 \tag{0.14}
$$

We recall that the ellipticity angle χ and the orientation angle ψ for the polarization ellipse are given, respectively, by

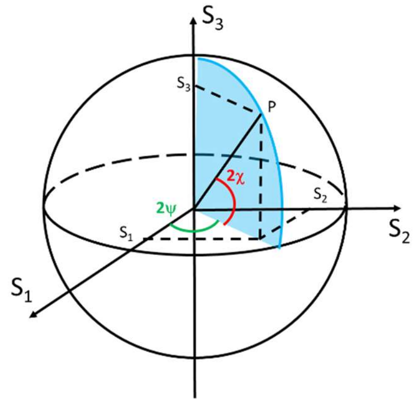


Figure 3-2.- Polarization State representation in the Poincaré Sphere

$$
\sin 2\chi = \frac{S_3}{S_0} \qquad -\frac{\pi}{4} \le \chi \le \frac{\pi}{4} \tag{0.15}
$$

$$
\tan 2\psi = \frac{S_2}{S_1} \qquad 0 \le \psi \le \pi \tag{0.16}
$$

Stokes vector

$$
S = \begin{pmatrix} E_{0x}^2 + E_{0y}^2 \\ E_{0x}^2 - E_{0y}^2 \\ 2E_{0x}E_{0y}\cos\delta \\ 2E_{0x}E_{0y}\sin\delta \end{pmatrix} = I_0 \begin{pmatrix} 1 \\ \cos 2\alpha \\ \sin 2\alpha \ \cos\delta \\ \sin 2\alpha \ \sin\delta \end{pmatrix} = S_0 \begin{pmatrix} 1 \\ \cos 2\chi \ \cos 2\psi \\ \cos 2\chi \ \sin 2\psi \\ \sin 2\chi \end{pmatrix} \tag{0.17}
$$

Linear Horizontally Polarized Light (LHP). For this case E0y = 0.

$$
\begin{aligned}
S_0 &= E_{0x}^2 \\
S_1 &= E_{0x}^2 \\
S_2 &= 0 \\
S_3 &= 0
\end{aligned} \tag{0.18}
$$

Linear Vertically Polarized Light (LVP). For this case E0x = 0.

$$
\begin{aligned}
S_0 &= E_{0y}^2 \\
S_1 &= -E_{0y}^2 \\
S_2 &= 0 \\
S_3 &= 0
\end{aligned}
\tag{0.19}
$$

Linear +45º Polarized Light (L+45). E0x = E0y = E0 and δ= 0.

$$
\begin{aligned}
S_0 &= 2E_0^2 \\
S_1 &= 0 \\
S_2 &= 2E_0^2 \\
S_3 &= 0
\end{aligned}
\tag{0.20}
$$

Linear -45º Polarized Light (L-45). E0x = E0y = E0 and δ= 180.

$$
\begin{aligned}
S_0 &= E_0^2 \\
S_1 &= 0 \\
S_2 &= -2E_0^2 \\
S_3 &= 0
\end{aligned}
\tag{0.21}
$$

Right Circularly Polarized Light (RCP). E0x = E0y = E0 and δ = 90º.

$$
\begin{aligned}
S_0 &= 2E_0^2 \\
S_1 &= 0 \\
S_2 &= 0 \\
S_3 &= 2E_0^2
\end{aligned}
\tag{0.22}
$$

Left Circularly Polarized Light (LCP). E0x = E0y = E0 and δ= -90º.

$$
\begin{aligned}
S_0 &= 2E_0^2 \\
S_1 &= 0 \\
S_2 &= 0 \\
S_3 &= -2E_0^2
\end{aligned}
\tag{0.23}
$$

4 POLARIZING DEVICES

Let us consider linear devices that do not change the plane wave structure of the wave but change the polarization state.

4.1 JONES FORMALISM

They can be treated as 2x2 matrixes that transform the input polarization states into the corresponding output polarization states: The Jones matrixes

$$J_2 = M \cdot J_1 \qquad \text{where} \qquad M = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} \tag{0.24}$$

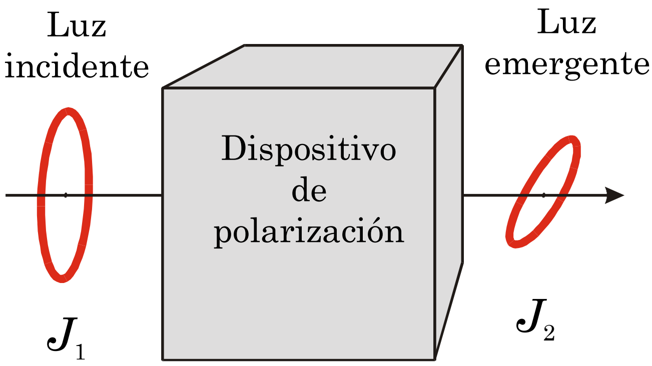


Figure 4-1 effect of a polarizing device on an input polarization beam

4.1.1 Linear Polarizer

$$M_P(0) = \begin{pmatrix} k_1 & 0 \\ 0 & k_2 \end{pmatrix} \tag{0.25}$$

Ideal

$$M_P(0) = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \tag{0.26}$$

Totally transmits one of the components (transmission axis), and totally absorbs the other component (absorption axis).

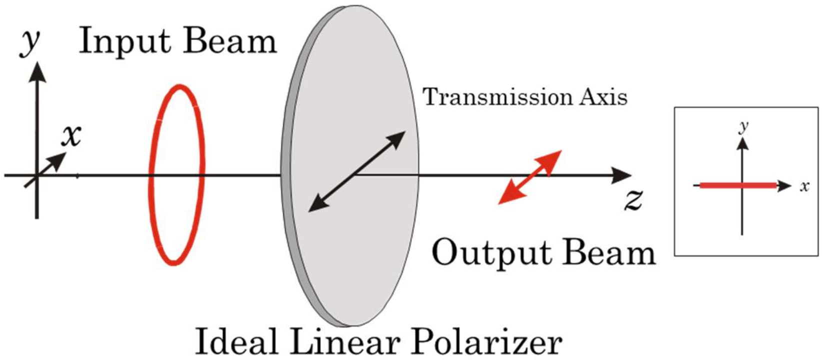


Figure 4-2 Effect of an ideal linear polarizer

$$J_{out} = A_x \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} A_x \\ A_y e^{j\delta} \end{pmatrix} \tag{0.27}$$

Malus Law

Input Light $J_1 = P_\theta = \begin{pmatrix} \cos(\theta) \\ \sin(\theta) \end{pmatrix}$ Exit Light $J_2 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \cdot J_1 = \cos(\theta) \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

4.1.2 Linear Wave-Plate

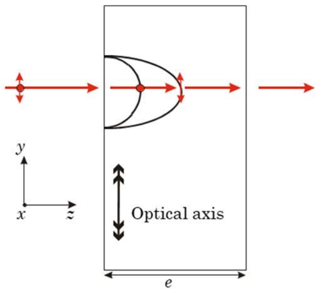


Figure 4-3 Waves in a wave-plate

Devices that introduce a phase shift between two orthogonal directions of the electric fields. Usually are anisotropic materials cutted with the optical axis parallel to the input face of the crystal. The ordinary and extraordinary waves travel with the same direction but with different speed, which produces the relative phase shift

Jones matrix when the direction of the optical axis coincides with one of the axes

$$M_D(\delta, 0) = \begin{pmatrix} 1 & 0 \\ 0 & e^{j\delta} \end{pmatrix} = e^{j\delta/2} \begin{pmatrix} e^{-j\delta/2} & 0 \\ 0 & e^{j\delta/2} \end{pmatrix} \tag{0.28}$$

$$\delta = \frac{2\pi}{\lambda_0}(n_e - n_o)e \tag{0.29}$$

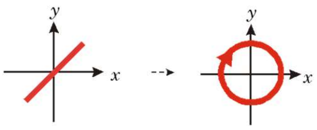
if $0<\delta<\pi$ slow axis => x axis.

In general, the wave retarders change the polarization state. Only the linear polarizations along the axis are not changed.

4.1.2.1 Quarter Wave plate: $\delta=\pi/2$

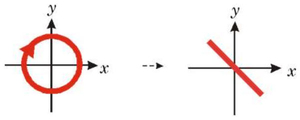
$$M_D(\pi/2,0)=\begin{pmatrix}1 & 0\\ 0 & j\end{pmatrix} \tag{0.30}$$

Changes linear polarizations at ±45º with respect to the neutral lines in circularly polarized light



$$J_1=\frac{1}{\sqrt{2}}\begin{pmatrix}1\\ \pm 1\end{pmatrix}\Rightarrow J_2=\frac{1}{\sqrt{2}}\begin{pmatrix}1\\ \pm j\end{pmatrix} \tag{0.31}$$

Changes circularly polarized light in linearly polarized light at ±45º with respect to the neutral lines

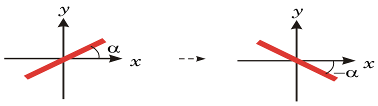


$$J_1=\frac{1}{\sqrt{2}}\begin{pmatrix}1\\ \pm j\end{pmatrix}\Rightarrow J_2=\frac{1}{\sqrt{2}}\begin{pmatrix}1\\ \mp 1\end{pmatrix} \tag{0.32}$$

4.1.2.2 Half Wave plate: $\delta=\pi$

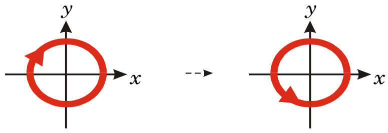
$$M_D(\pi,0)=\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix} \tag{0.33}$$

Changes linear polarizations at αº with respect to the neutral lines in linear polarizations at -αº



$$J_{in}=\begin{pmatrix}\cos(\alpha)\\ \sin(\alpha)\end{pmatrix}\Rightarrow J_{out}=\begin{pmatrix}\cos(\alpha)\\ -\sin(\alpha)\end{pmatrix}=\begin{pmatrix}\cos(-\alpha)\\ \sin(-\alpha)\end{pmatrix} \tag{0.34}$$

Changes (R/L) circular light in (L/R) circular light



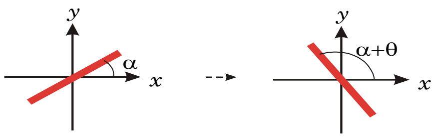
$$J_1=\begin{pmatrix}1\\ \pm j\end{pmatrix}\Rightarrow J_2=\begin{pmatrix}1\\ \mp j\end{pmatrix} \tag{0.35}$$

4.1.3 Polarization Rotators

Rotate the polarization ellipse an angle θ

$$M_R(\theta)=\begin{pmatrix}\cos(\theta) & -\sin(\theta)\\ \sin(\theta) & \cos(\theta)\end{pmatrix} \tag{0.36}$$

Example



$$J_1=\begin{pmatrix}\cos(\alpha)\\ \sin(\alpha)\end{pmatrix}\Rightarrow J_2=\begin{pmatrix}\cos(\alpha+\theta)\\ \sin(\alpha+\theta)\end{pmatrix} \tag{0.37}$$

4.1.4 Coordinates Transformation

The above elements may present a rotation on the x,y plane. Then if we know the Jones matrix in a coordinate system, what is the Jones matrix in a rotated coordinate system?

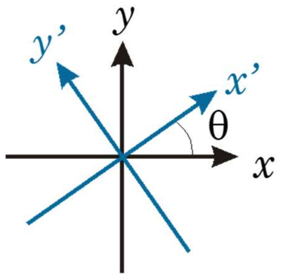


Figure 4-4 Coordinates transformation

$$M = R(-\theta)\cdot M' \cdot R(+\theta) \tag{0.38}$$

$$R(+\theta) = \begin{pmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{pmatrix} \tag{0.39}$$

Example: Linear polarizer rotated an angle θ.

$$M_P(\theta) = R(-\theta)\cdot\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\cdot R(+\theta) = \begin{pmatrix} \cos^2(\theta) & \cos(\theta)\sin(\theta) \\ \cos(\theta)\sin(\theta) & \sin^2(\theta) \end{pmatrix} \tag{0.40}$$

$$M_P(\theta = 90^\circ) = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

4.1.4.1 Cascaded Polarization Devices

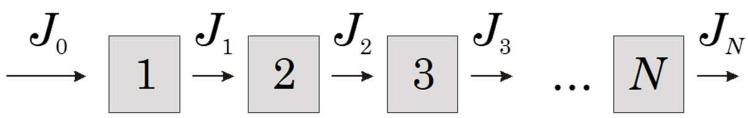


Figure 4-5Effect of cascaded polarizing devices

$$J_N = M_N \cdot \ldots M_3 \cdot M_2 \cdot M_1 \cdot J_0 \tag{0.41}$$

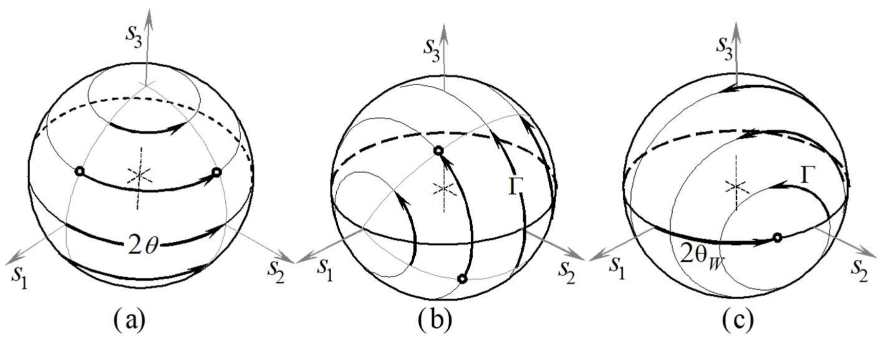


Figure 4-6. Representation on the Poincaré sphere of the polarization transformations produced by:

(a) rotators,
(b) Linear Phase Shifters with the neutral axis on the x,y axis
(c) Linear Phase Shifters with the neutral axis in an arbitrary direction (θ_w)

4.2 Mueller Formalism

$$\begin{pmatrix} S'_0 \\ S'_1 \\ S'_2 \\ S'_3 \end{pmatrix} = \begin{pmatrix} m_{00} & m_{01} & m_{02} & m_{03} \\ m_{10} & m_{11} & m_{12} & m_{13} \\ m_{20} & m_{21} & m_{22} & m_{23} \\ m_{30} & m_{31} & m_{32} & m_{33} \end{pmatrix} \begin{pmatrix} S_0 \\ S_1 \\ S_2 \\ S_3 \end{pmatrix} \quad S' = M.S \tag{0.42}$$

When an optical beam interacts with matter its polarization state is almost always changed. In fact, this appears to be the rule rather than the exception. The polarization state can be changed by (1) changing the amplitudes, (2) changing the phase, (3) changing the direction of the orthogonal field components, or (4) transferring energy from polarized states to the unpolarized state.

4.2.1 Linear Polarizer

$$M = \frac{1}{2} \begin{pmatrix} p_x^2 + p_y^2 & p_x^2 - p_y^2 & 0 & 0 \\ p_x^2 - p_y^2 & p_x^2 + p_y^2 & 0 & 0 \\ 0 & 0 & 2p_x p_y & 0 \\ 0 & 0 & 0 & 2p_x p_y \end{pmatrix} \quad 0 \le p_{x,y} \le 1 \tag{0.43}$$

For a neutral density filter px = py = p

$$M = p^2 \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{0.44}$$

perfect linear polarizer if the transmission factor px is unity

$$M = \frac{1}{2} \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{0.45}$$

4.2.2 Retarder

$$M = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cos\phi & \sin\phi \\ 0 & 0 & -\sin\phi & \cos\phi \end{pmatrix} \tag{0.46}$$

For a quarter-wave retarder

$$M = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix} \tag{0.47}$$

half-wave retarder

$$M = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \tag{0.48}$$

4.2.3 The Mueller Matrix of a Rotator

$$M\left(2\theta\right) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos 2\theta & \sin 2\theta & 0 \\ 0 & -\sin 2\theta & \cos 2\theta & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{0.49}$$

We note that a physical rotation of θ leads to the appearance of 2 θ rather than θ because we are working in the intensity domain; in the amplitude domain we would expect just θ.

4.2.4 Mueller Matrices for Rotated Polarizing Components

$$M\left(2\theta\right) = M_R\left(-2\theta\right) M M_R\left(2\theta\right) \tag{0.50}$$

Mueller matrix for an ideal linear rotated polarizer

$$M_P\left(2\theta\right) = \begin{pmatrix} 1 & \cos 2\theta & \sin 2\theta & 0 \\ \cos 2\theta & \cos^2 2\theta & \sin 2\theta \cos 2\theta & 0 \\ \sin 2\theta & \sin 2\theta \cos 2\theta & \sin^2 2\theta & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{0.51}$$

Mueller matrix for the rotated retarder

$$M_R\left(\phi, 2\theta\right) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos^2 2\theta + \cos\phi \sin^2 2\theta & \left(1-\cos\phi\right)\sin 2\theta \cos 2\theta & -\sin\phi \sin 2\theta \\ 0 & \left(1-\cos\phi\right)\sin 2\theta \cos 2\theta & \sin^2 2\theta + \cos\phi \cos^2 2\theta & \sin\phi \cos 2\theta \\ 0 & \sin\phi \sin 2\theta & -\sin\phi \cos 2\theta & \cos\phi \end{pmatrix} \tag{0.52}$$

4.3 DETERMINATION OF THE POLARIZATION TYPE

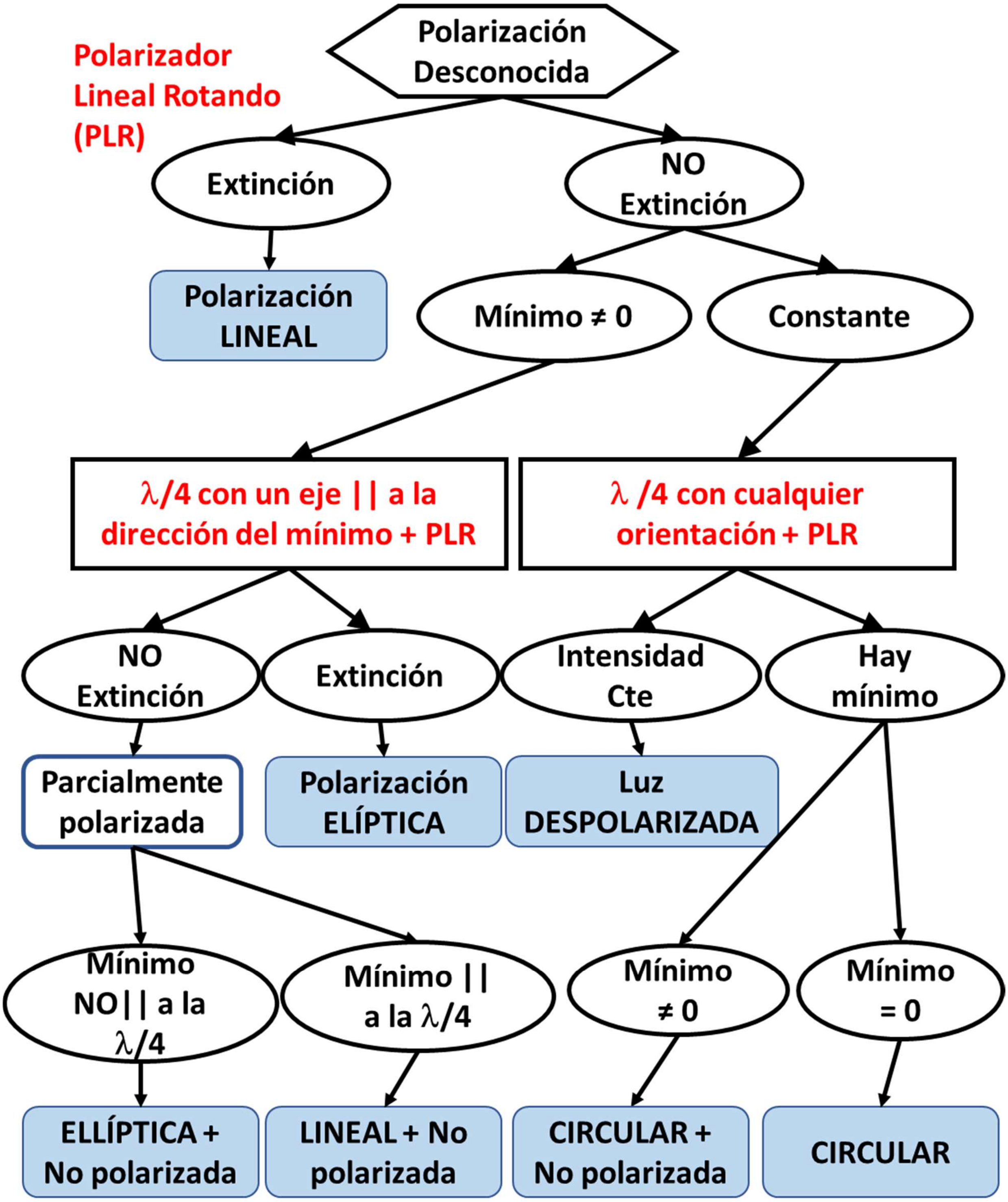


Figure 4-7. Determination of the polarization Type

4.4 SPECIAL DEVICES

4.4.1 Lyot-Öhman filter

A series of N parallel crystal plates are interspersed with polarizers and the retardation of each crystal is twice that of the preceding one. The crystals are aligned so that their axes are parallel to one another. The planes of polarization of the polarizers are inclined at 45◦ to the axes. It can be shown that the spectral transmittance of such a filter is given by the expression

$$T = \frac{1}{2} T_0 \left(\frac{\sin 2^N \pi \xi_1}{2^N \pi \xi_1} \right)^2 \tag{0.53}$$

Here ξ_1 is the retardation of the thinnest component. T_0 is a factor that allows for the light losses (mostly in the polarizers). Equation (21) represents a series of transmission peaks of half-width λ/(2Nξ1) that are separated from one another by a wavelength interval λ/ξ1. Light transmitted between any two adjacent peaks amounts to about 11% of the transmission within one peak.

The largest stage sets the bandwidth, and the smallest stage sets the Free Spectral Range. If you use two of the second largest crystals, it will increase the contrast of the desired line. If you split the crystals in half and add a 1/2 waveplates in the middle, you can increase the field of view of the filter. The separation and narrowness of the transmission peaks depends on the number, thicknesses, and orientation of the plates.

4.4.2 Solc filter

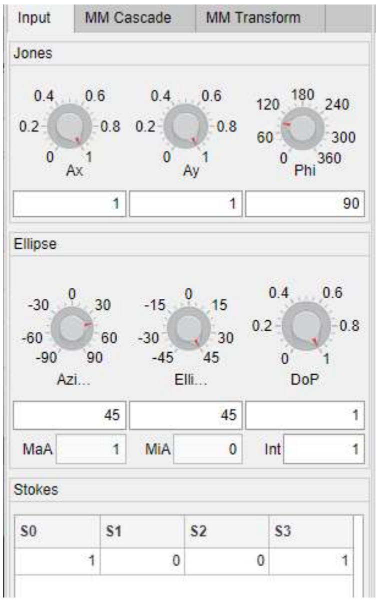
It is composed of N identical birefringent crystal plates placed between two polarizers. Two variants of this filter exist. In the fan configuration the two polarizers are parallel to one another and the ith plate is inclined at an angle (i − 0.5)π/2N to them.

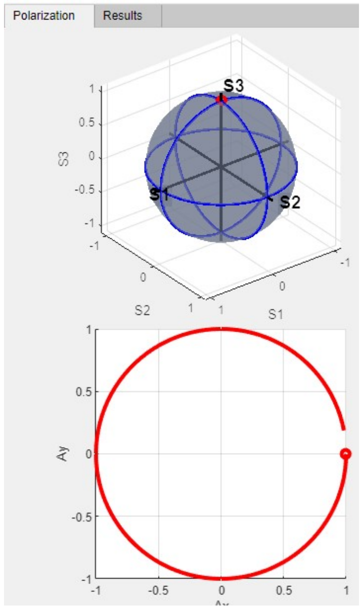
The polarizers in the folded configuration are crossed, and the ith plate makes an angle $(-1)^i\pi/4N$ with the first polarizer. The spectral transmittances of both configurations resemble the transmittances of Lyot- Öhman filters of similar overall thickness and with the thinnest crystal plate of same thickness as those of the components of the Solc filter. The main departures are that T_0 for the Solc filters is about 0.72, which is much higher than that of Lyot- Öhman filters, and that the parasitic transmission is about 2.5 times as high

5 USER INTERFACE

When the program starts, the main window appears. On the left are the tabs where you can enter the data and launch the different calculations. Results are shown on the right

5.1 DEFINITION OF INITIAL POLARIZATION

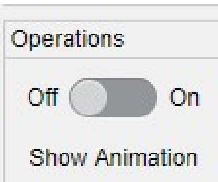




In the "Input" tab there are the different ways to enter the polarization state:

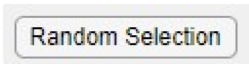
- Jones vector
- Polarization ellipse
- Stokes parameters

When any of the parameters of one of the representations is changed, the other representations are calculated. In addition, the polarization ellipse and the representation in the Poincaré sphere are shown



If the option to show animation is selected, a new window appears with the wave propagation and the polarization ellipse as a function of time

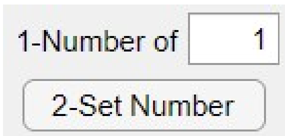
If the check box “RT” is selected, then the animation tries to change the time in real time. If not, each time step of the animation is the same.



By pressing the “Random Selection” button a random polarization state is generated

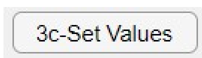
By pressing the "Init Sphere" button, all the states drawn in the Poincaré sphere are deleted.

5.2 CHANGE OF POLARIZATION BY A SET OF DEVICES



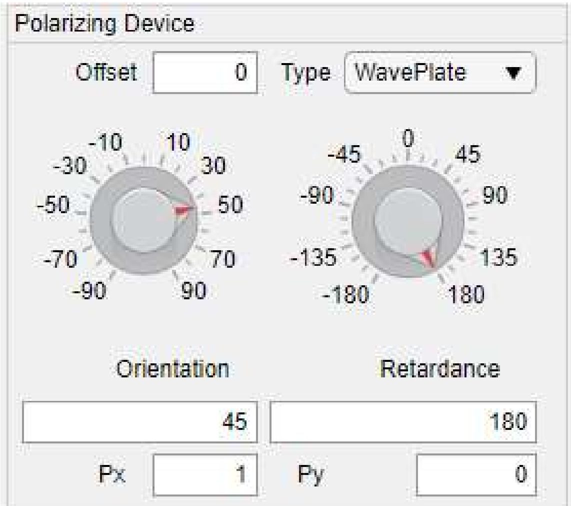
You can define a set of devices that act in cascade in the "MM Cascade" tab. The steps to follow are:

0. The input polarization is the one defined in the previous tab "Input"
1. First, we must define the number of devices that we are going to use by entering “Number of” and
2. pressing the “Set Number” button.
3. Next, you must define the elements. For each of the elements you must select
   a. Element number to define
   b. Element definition (see next section)
   c. Set the values by pressing the "Set Values" button. BEWARE you have to do it for each element.



b) DEFINITION OF THE POLARIZING ELEMENT. It is done in the box "Polarizing Device"

b1.- Select type: Polarizer, Wave Plate, or Rotator



b2.- Select the orientation, either on the dial or in the corresponding box. The Offset is added to this orientation (simulation of an offset with respect to the axis of the device that supports the element).

b3.- If it is a not perfect polarizer, give the transmissions along the axes X and Y: Px, Py.

b4.- If it is a wave plate, give the delay, either on the dial or in the corresponding box.

You can also define the set of devices automatically by selecting one of the predefined sets in "Set-Up". The options that exist are:

1 polarizer, 2 polarizers, 3 polarizers, Polarizer-WavePlate-Polarizer.

Once the Set-Up has been selected, each of the elements must be defined as explained in b).

There are 3 possible sets of outputs

1. Polarization change
2. Output Intensity as a function of a parameter
3. Spectral dependence.

In cases 1) and 2) you can change the parameter

a) Orientation of the device selected in the "Element" box

b) Delay of the device selected in the "Element" box (in the case of a sheet)

In case 3) we assume that what changes is the device delay as a function of wavelength. The delay is 2pi (n * e)/ lambda. We will assume that the index n does not depend on the wavelength, and the only change comes from the lambda in the denominator. You can change the maximum delay of the delay dial in the “Max Ret” box. With which thick sheets can be simulated. The delay that is selected on the dial will be that of the initial lambda, then it decreases as the lambda increases. Assuming that the input composition of the light is white, with the same intensity in all lambdas, the approximate “color” at the output would be shown in the upper graph.

5.3 SIMULATION OF A SET OF SCOTCH

In the last tab "MM Transforms" there is a simulation of different zones. Each has a different set of retarder sheets with maximum delay given in “Retardance”. Each one has a different combination of up to 5 layers with different orientations and the same delay”.

The transmission is calculated as a function of the wavelength and the distribution of approximate colors that would be obtained when illuminating with the initial polarization and a polarizer at the output with the given orientation is drawn.

6 EXERCISES

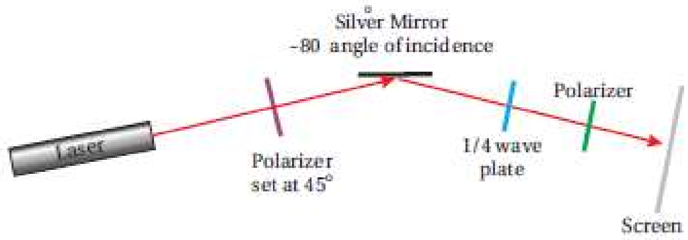
1) Play with the values of E_x, E_y, φ (amplitudes of both components and phase difference) and see the evolution of the ellipse. Pay special attention when $\varphi = 0,\ \ \pi,\ \ \pi/2,\ \ 3\pi/2$ in the last two cases, what happens if both amplitudes are equal?
2) Show that the addition of two circular polarization states R and L with the same amplitude and a phase difference d, is a linear polarization state rotated half of the phase difference
3) Generate a linear polarization state with a given azimuth. In the tab “MM transformations” a) select the retardance of the wave plate (for instance 90º) b) change the Retarder Orientation and NOTE that when it coincides with the azimuth or perpendicular to the azimuth the output polarization state is the same as the input one. Note that at these orientations the output polarization state is independent of the wave-plate retardance (change the retardance)
4) Demonstrate that two $\lambda/2$ wave-plates act as a polarization rotator of an angle equal to twice the angle between both. Use the polarization applet (PA) to visualize this. First select an input polarization state in the tab “Input Polarization”, then, in the tab “MMtransformations”, set the retardance of both wave-plates to 180º, select the button “Activate 2nd retarder” and change the Orientation of one of the retarder. You will visualize the output Stokes, Azimuth, and ellipticity, and you can see the polarization ellipse and the Poincaré Sphere. If the button “Continuous Draw” is on, then you can see the evolution of the output polarization state. Does the Ellipticity change?
5) Demonstrate that with a linear polarizer at 0º and two variable wave-plates oriented at 45º and at 0º respectively it is possible to obtain any polarization state. What are the retardances needed to obtain a given Stokes vector? NOTE that the solution is not unique when the wave-plates retardation range is $\left[0-360\right]$.

   Use the tab “MM transformations”, set the retarder orientations, set the “activate de 2nd retarder2” button to on and the “Continuous Draw”. Change the retardance of the first retarder gradually until arrive to the desired retardance, and then make the same with the second retarder to visualize the trajectory of the polarization on the Poincaré Sphere. Try to obtain the following output polarization states:
   a) Linear at 45º, Linear at 22.5º, … Note that in this case, in one of the solutions, the retardance of the first retarder should be twice the final azimuth, and the retardance of the second retarder should be 90º.
   b) ….
6) Let us assume that we have a linear polarizer and a wave plate. How can we obtain circularly polarized light? Is it possible to obtain it with arbitrary retardance of the wave plate? Show this with the applet.
7) Let us assume we have a linear polarizer and a variable wave-plate that we can rotate. Is it possible to obtain any polarization state? If yes, what should be the orientation of both devices and retardance of the wave-plate? Implement this in the Polarization applet.
8) Let us assume that we have two linear polarizers and a quarter-wave plate, but we do not know the location of the respective axis. We can measure the light intensity but not its polarization. Describe a method to obtain circularly polarized light. Implement it in the polarization applet.
9) Generate a polarization state randomly. Use the protocol shown in **¡Error! No se encuentra el origen de la referencia.** to determine the polarization type.
10) A quarter wave plate is rotated between two crossed Polaroids. If an un-polarized beam is incident on the first Polaroid, discuss the variation of intensity of the emergent beam as the quarter wave plate is rotated. What will happen if we have a half wave instead of a quarter wave plate?
11) (a) Consider two crossed Polaroids placed in the path of an unpolarized beam of intensity I0 (see Fig. 22.6). If we place a third Polaroid in between the two, then, in general, some light will be transmitted through. Explain this phenomenon. (b) Assuming the pass axis of the third Polaroid to be at 45° to the pass axis of either of the Polaroids, calculate the intensity of the transmitted beam. Assume that all the Polaroids are perfect.

12) Consider we have two crossed polarizers and a variable wave-plate between them. (a) Evaluate the output irradiance as a function of the retardance. (b) What should be the wave-plate orientation to obtain the maximum contrast? (c) Implement this set-up in the polarization applet.

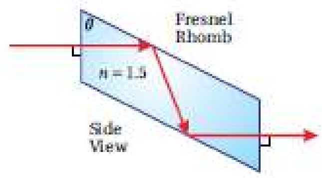
13) Design a Lyot filter win a bandwidth of 0.1 nm at λ=656.3nm. Find the required thickness of the thickest plate.

14) [3] Create a source of unknown elliptical polarization by reflecting a linearly polarized laser beam (with both s and p-components) from a metal mirror with a large incident angle (i.e. α>80º). Use a quarterwave plate and a polarizer to determine the Jones vector of the reflected beam. Find the ellipticity, the helicity (right or left handed), and the orientation of the major axis. HINT: A polarizer alone can reveal the direction of the major and minor axes and the ellipticity, but it does not reveal the helicity. Use a quarterwave plate (oriented at a special angle μ) to convert the unknown elliptically polarized light into linearly polarized light. A subsequent polarizer can then extinguish the light, from which you can determine the Jones vector of the light coming through the wave plate. This must equal the original (unknown) Jones vector (6.11) operated on by the wave plate (6.37). As you solve the matrix equation, it is helpful to note that the inverse of (6.37) is its own complex conjugate.



15) [3] Un haz de luz linearmente polarizado a 45º es reflejado por un espejo de plata con un ángulo de incidencia de 80º (como en el problema anterior). ¿Cuál es el vector de Jones de la luz reflejada?

16) [3] Calcule el ángulo de corte θ de un vidrio en un rombo de Fresnel para que después de dos reflexiones haya una diferencia de fase de $\pi/2$ entre los dos estados de polarización, de tal manera que actúe como una lámina de cuarto de onda. NOTA: la solución analítica puede ser complicada. Haga un dibujo para encontrar una solución numérica. SOLUCIÓN: hay dosánguloa que funcionan θ=50º, y 53º



17) [3] Una manera de construir un polarizador dircular dextrógiro es mediante una lámina $\lambda/4$ con el eje rápido a 45º, seguida de un polarizador lineal orientado vertical y de una lámina de $\lambda/4$ con el eje rápido a -45º. Calcule la matriz de Jones del dispositivo. Copruebe que este dispositivo no cambia la luz polarizada circular dextrógira, mientras que absorbe la luz polarizada circular levógira.